Wind Resistant Design of Long Span Bridges

No.5

--- Gust Response Analysis ---

Sungkyunkwan University

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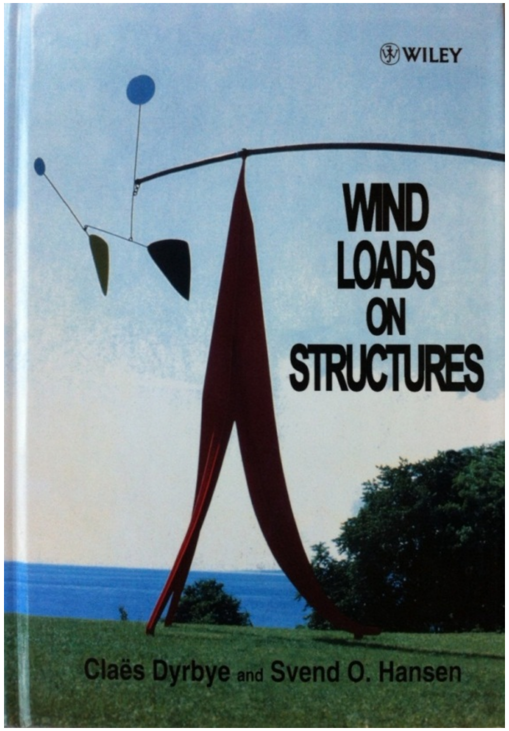
Method-1

① WIND LOADS ON STRUCTERE

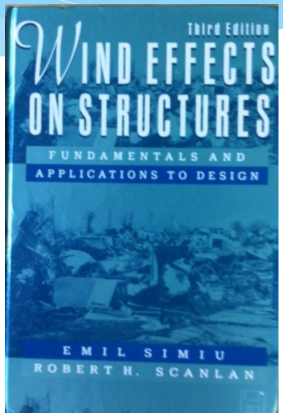
//Gust response analysis here is from this book.

I recommend to read this book.

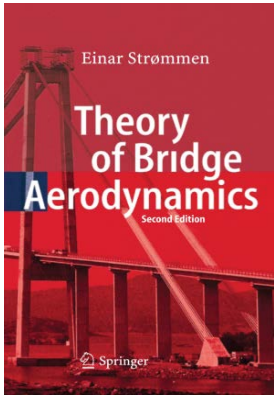


Method-2

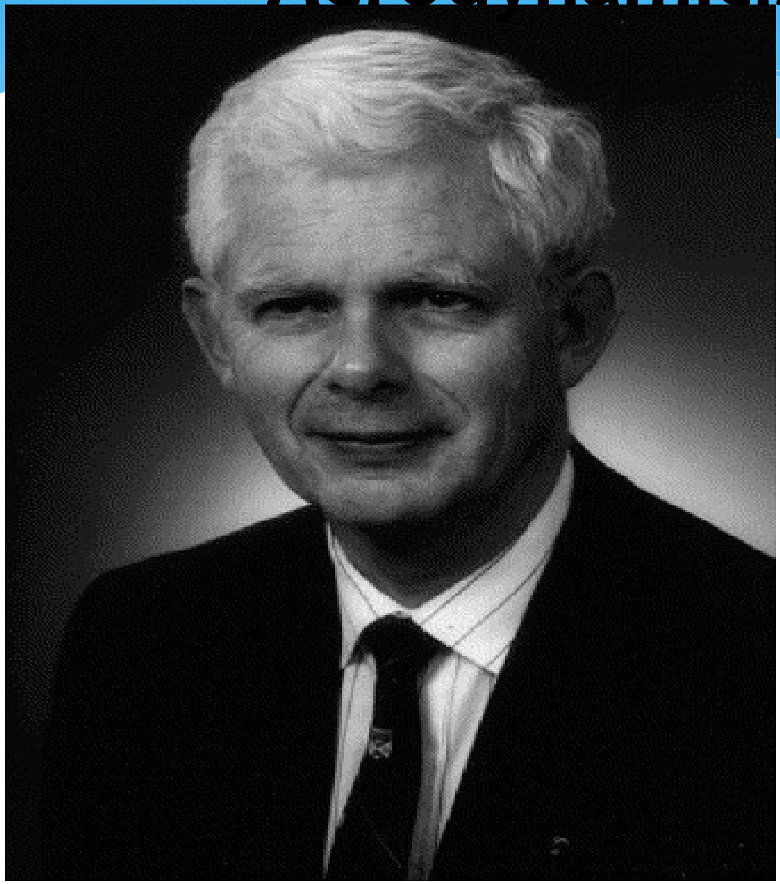
② Gust response analysis is based upon Prof. Scanlan's Method.
// His method is simple and easy by hands.



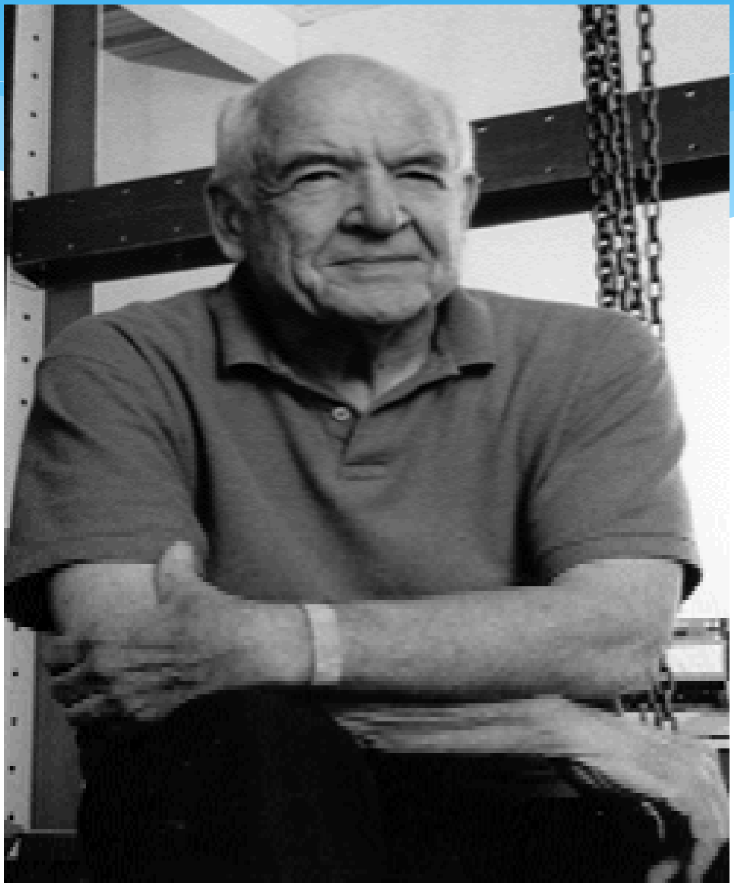
③ Einar Strommen: Theory of Bridge Aerodynamics
Second Edition Chapter 6
// Theoretical base is detailed.



Great Bridge Aerodynamicist



A. G. Davenport
（Gust Response Analysis）



R. H. Scanlan
（Flutter Analysis）

Random Process (1)

- A random process that is a function of time is called *stationary* if its statistical properties are not depend upon the choice of the time origin.
- A function belonging to an ensemble that might be generated by a stationary random process, or a stationary random signal, is thus assumed to extend over the entire time domain. Its mean and
- its mean square value do not vary with time (see Fig. after page).

Random Process (2)

- The ensemble average, other expectation, of a random process is the average of the values of the member functions at any particular time.
- A stationary random process is said to be ergodic if, for that process, time averages equal ensemble averages. Ergodicity requires in effect that every sample function be typical of the entire ensemble (Fig. after page).

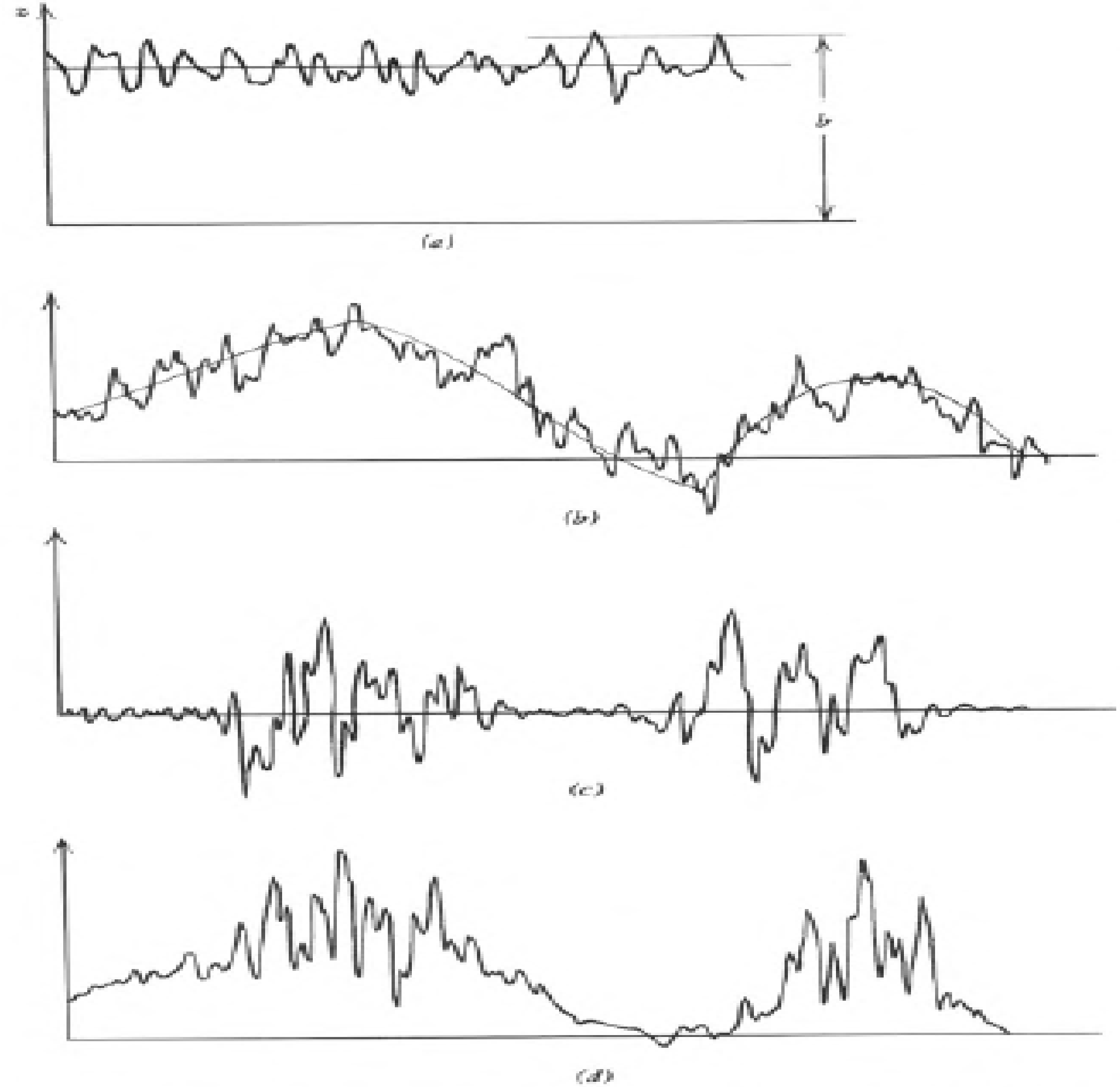
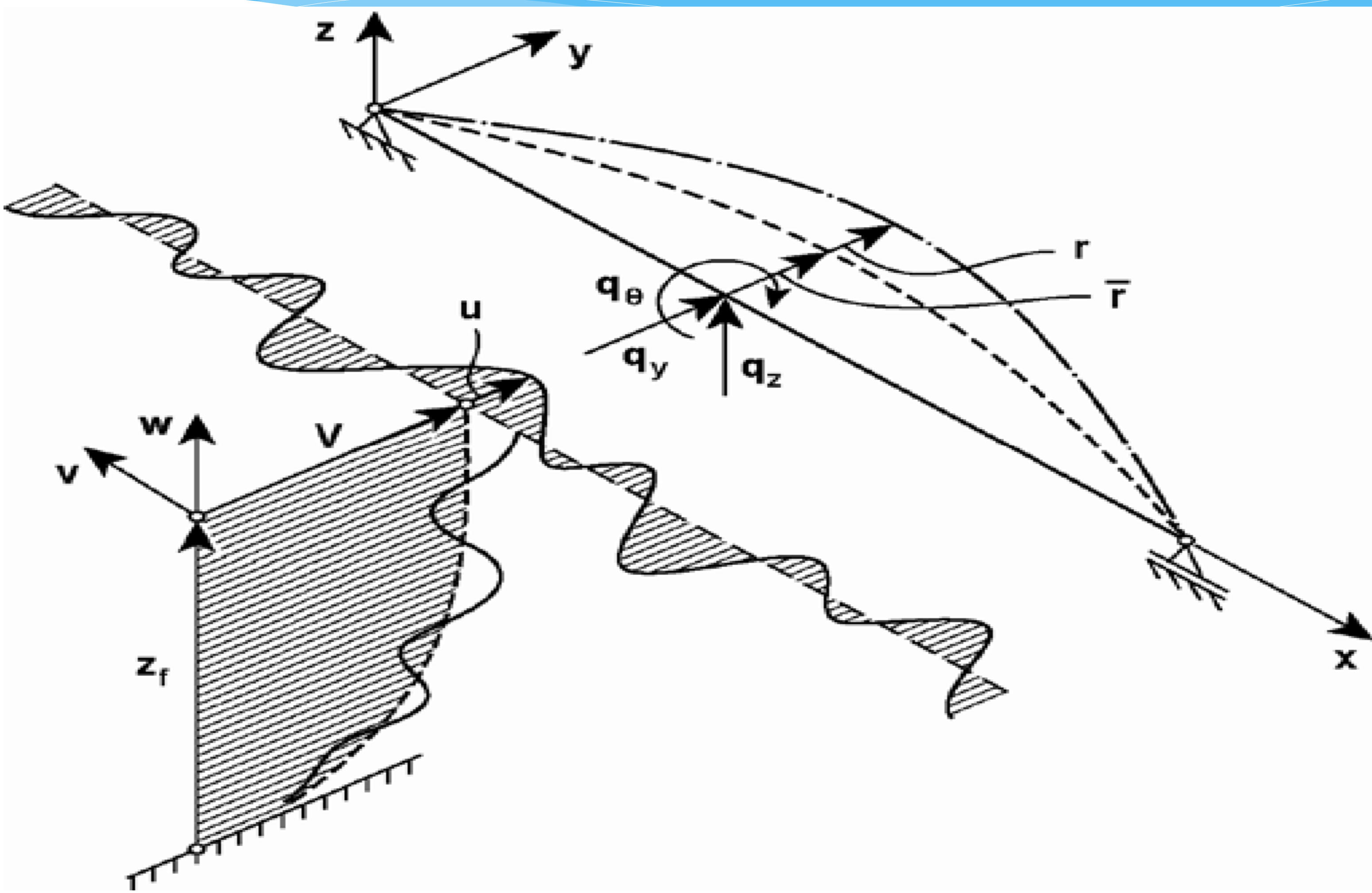
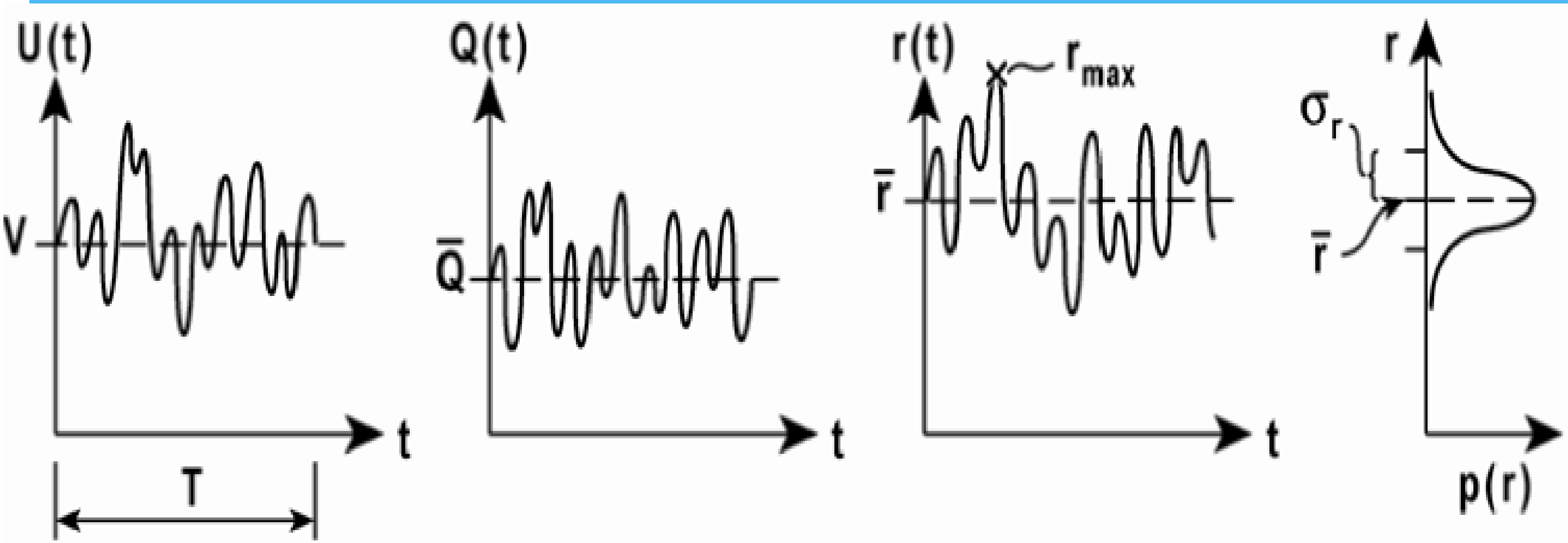


FIGURE A2.1. (*a*) Stationary signal. Nonstationary signals with: (*b*) time varying mean value; (*c*) time varying mean square value; (*d*) time varying mean and mean square value. After J. S. Bendat and A. G. Piersol, *Random Data: Analysis and Measurement Procedures*, Wiley-Interscience, New York, 1971, p. 345.

Simple bridge structural system subject to fluctuating wind field

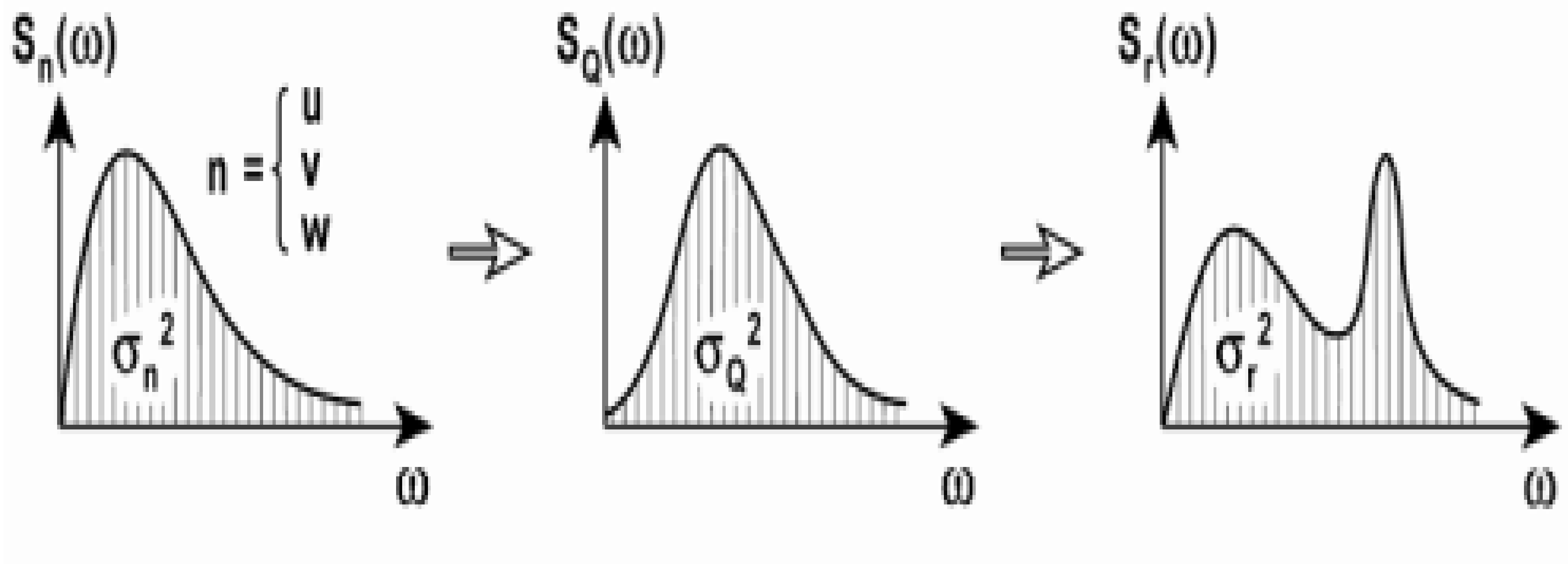


Time domain



Wind Fluctuation **Member Force** **Displacement** **Distribution**

Frequency domain



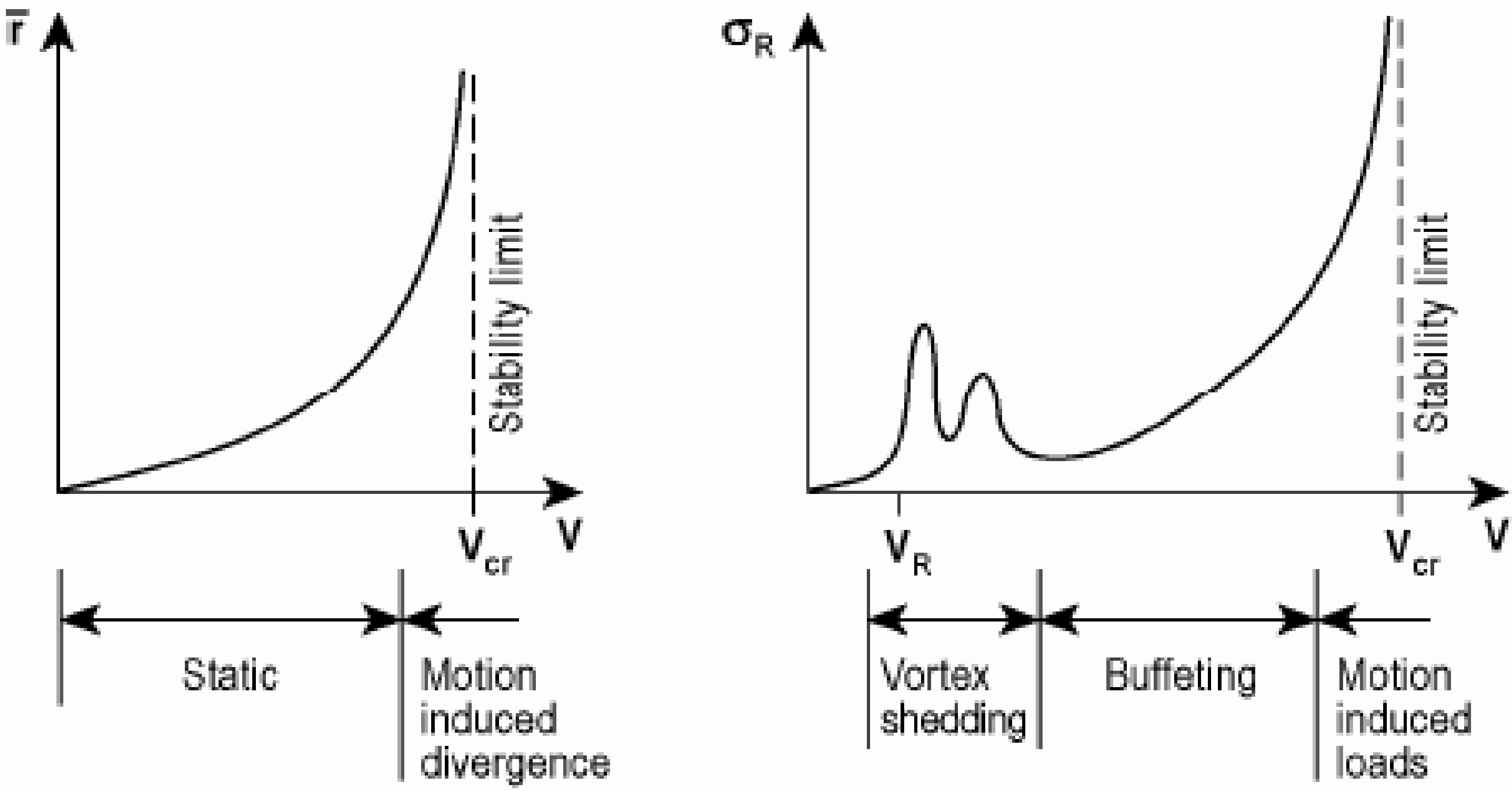
PSD of Wind Gust

PSD of Member Force

PSD of Displacement

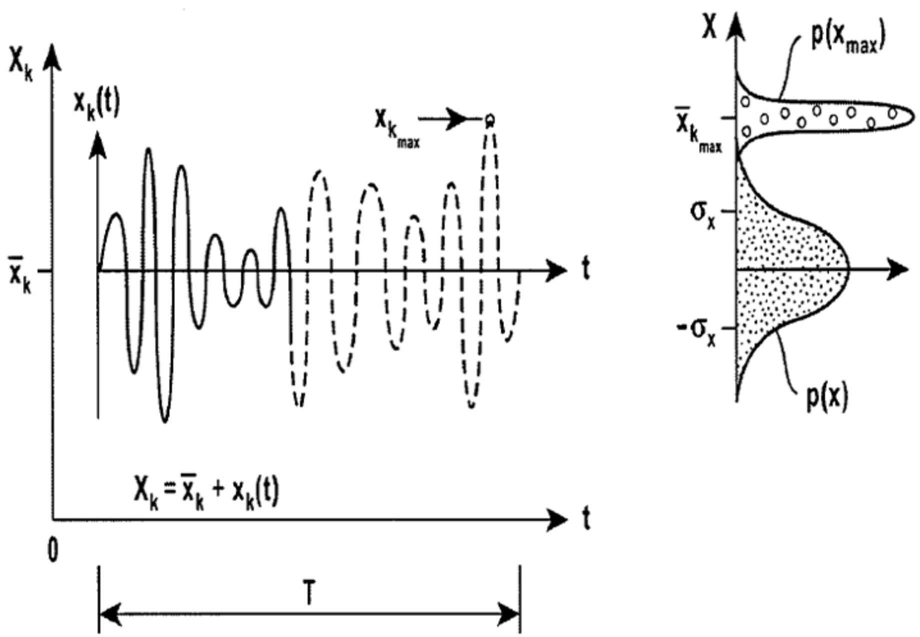
NB) Power Spectral Density (PSD)

Typical response variation with mean wind velocity

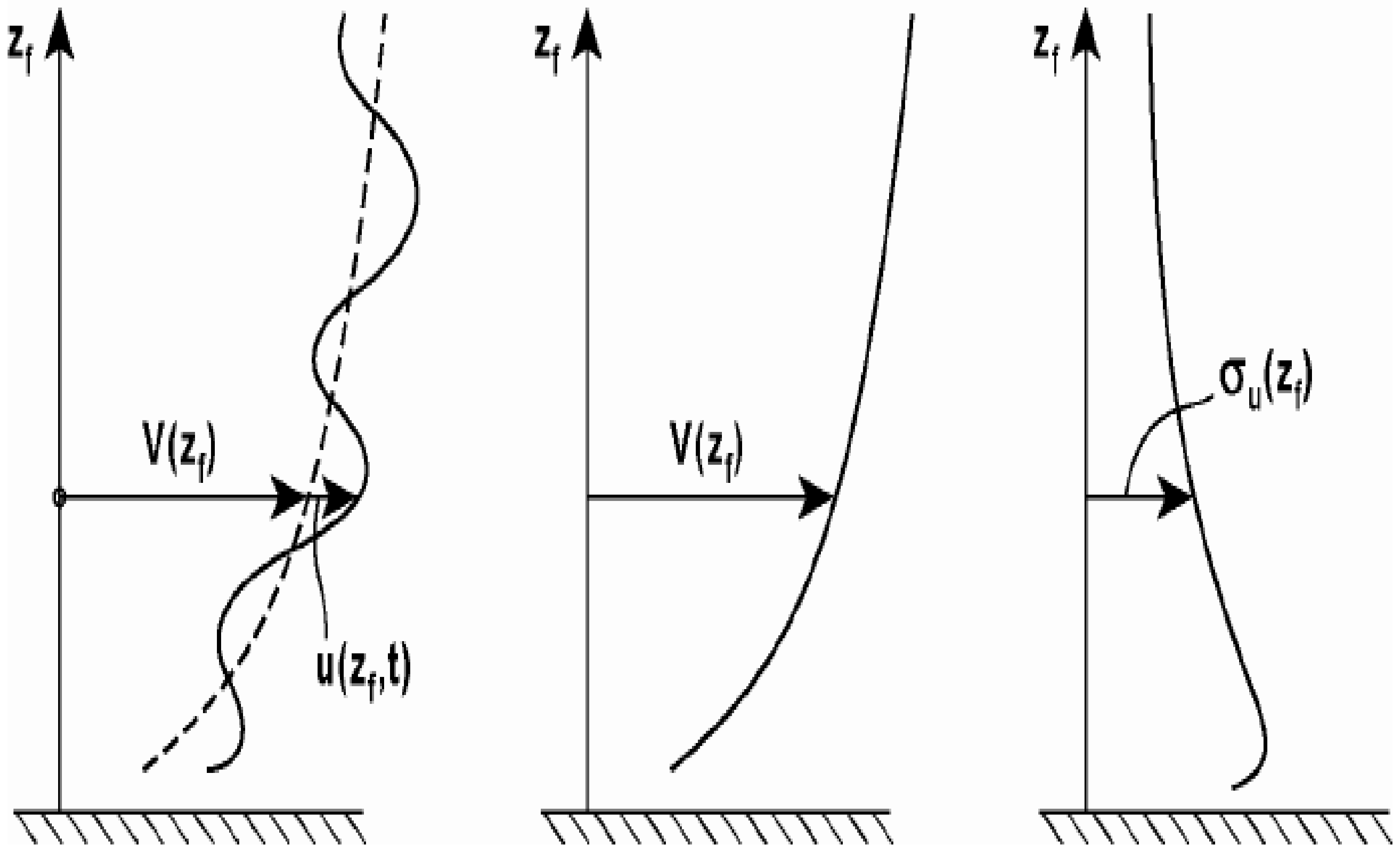


Short Term Stationary Random Process

$q_u(t)$:

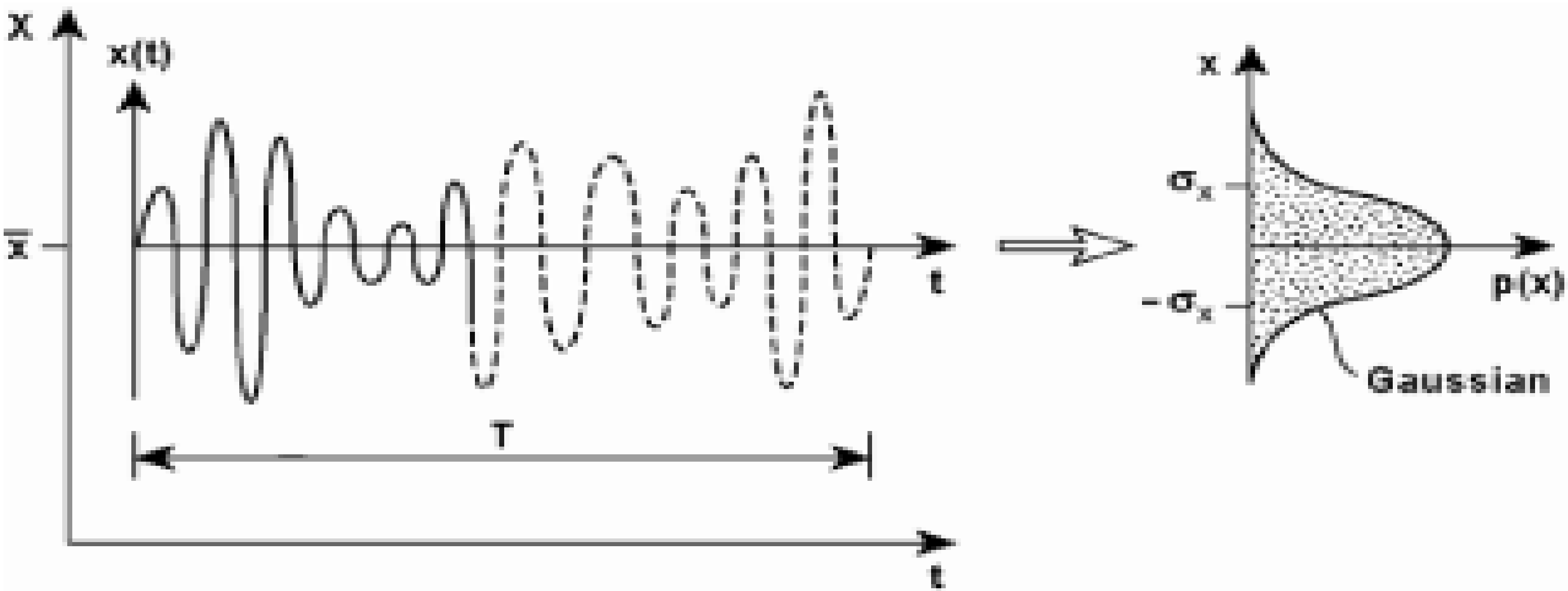


The wind velocity and turbulent profile



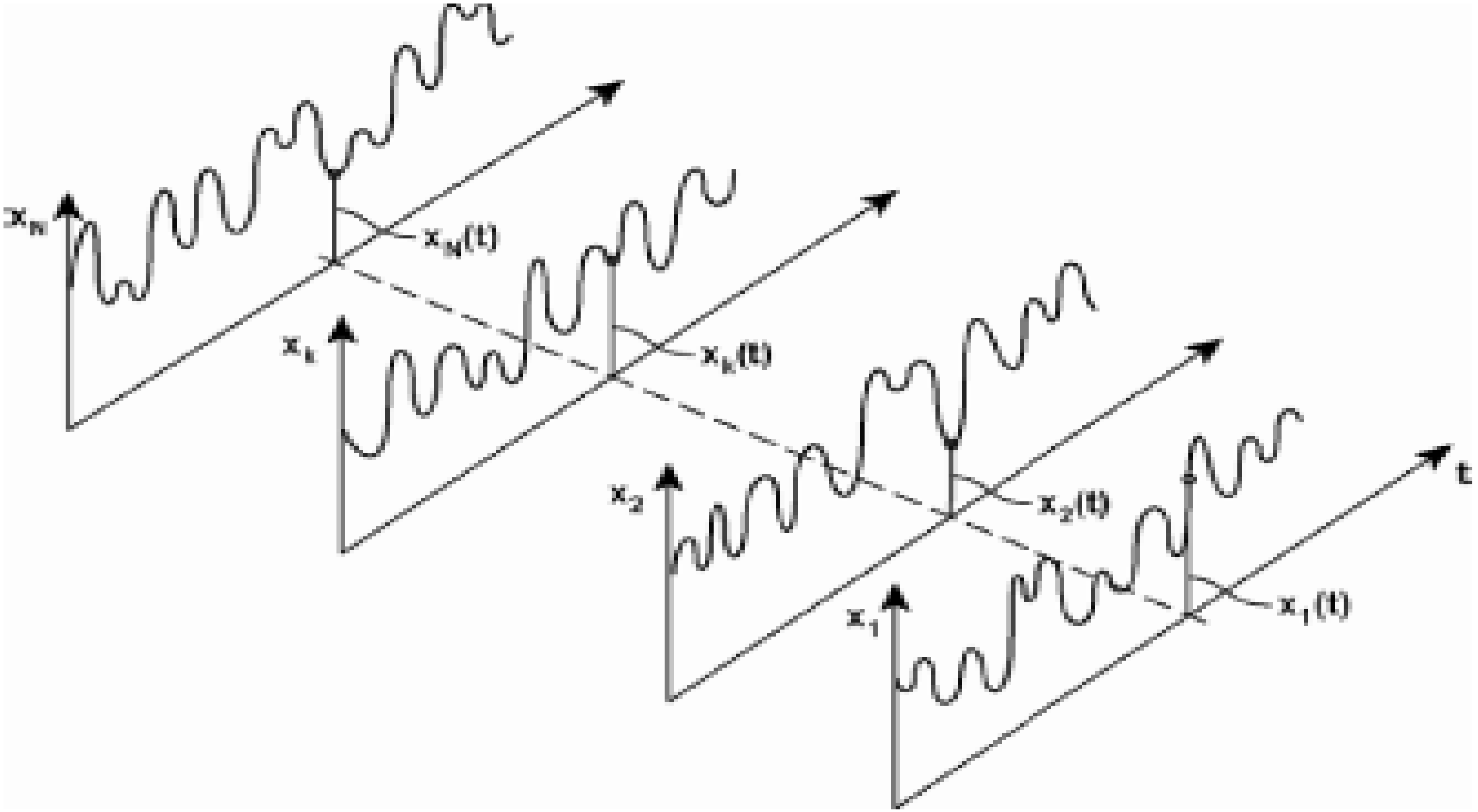
Gusty Wind Mean Velocity Variance

Time domain statistics

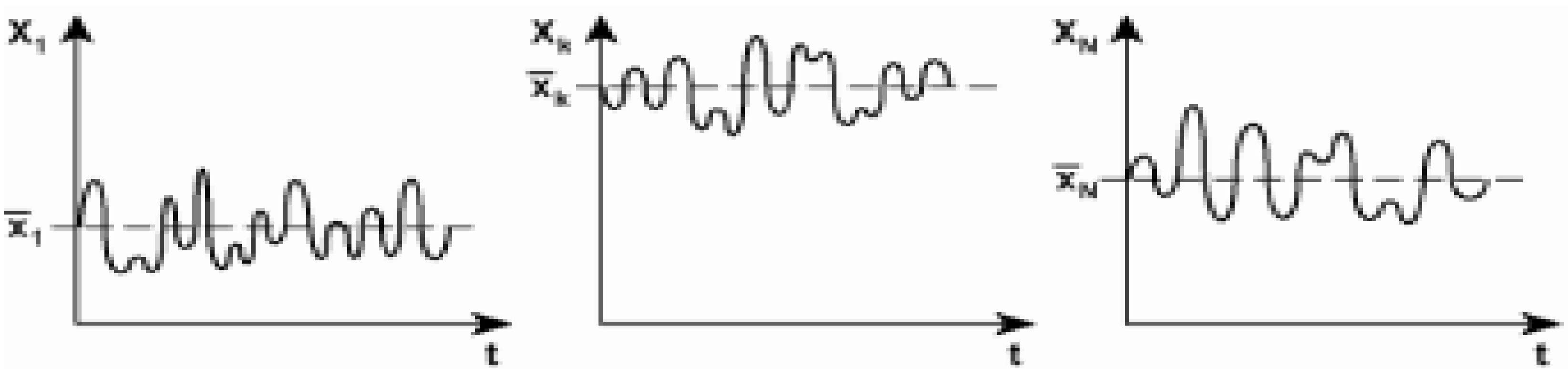


$$\left.\begin{aligned} \bar{x} &= \lim_{T\to\infty}\frac{1}{T}\int_0^T X\,dt \\ \sigma_x^2 &= \lim_{T\to\infty}\frac{1}{T}\int_0^T [x(t)]^2\,dt \end{aligned}\right\} \qquad (2.9)$$

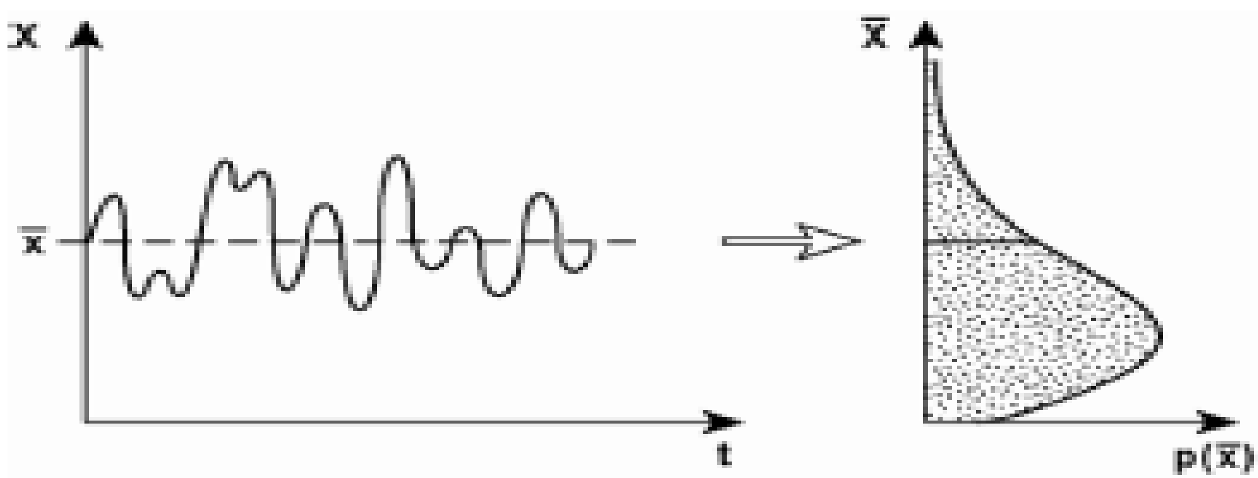
Ensemble statistics of simultaneous events



The probability of mean values

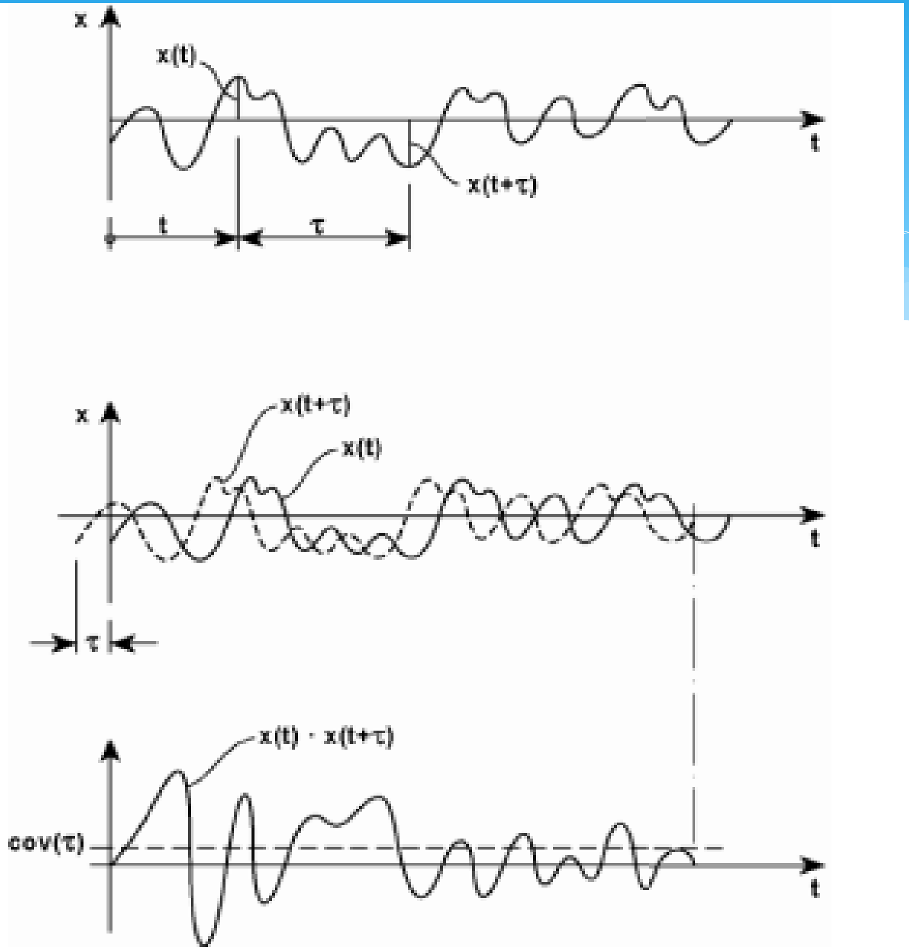


a) *N independent short term realisations*



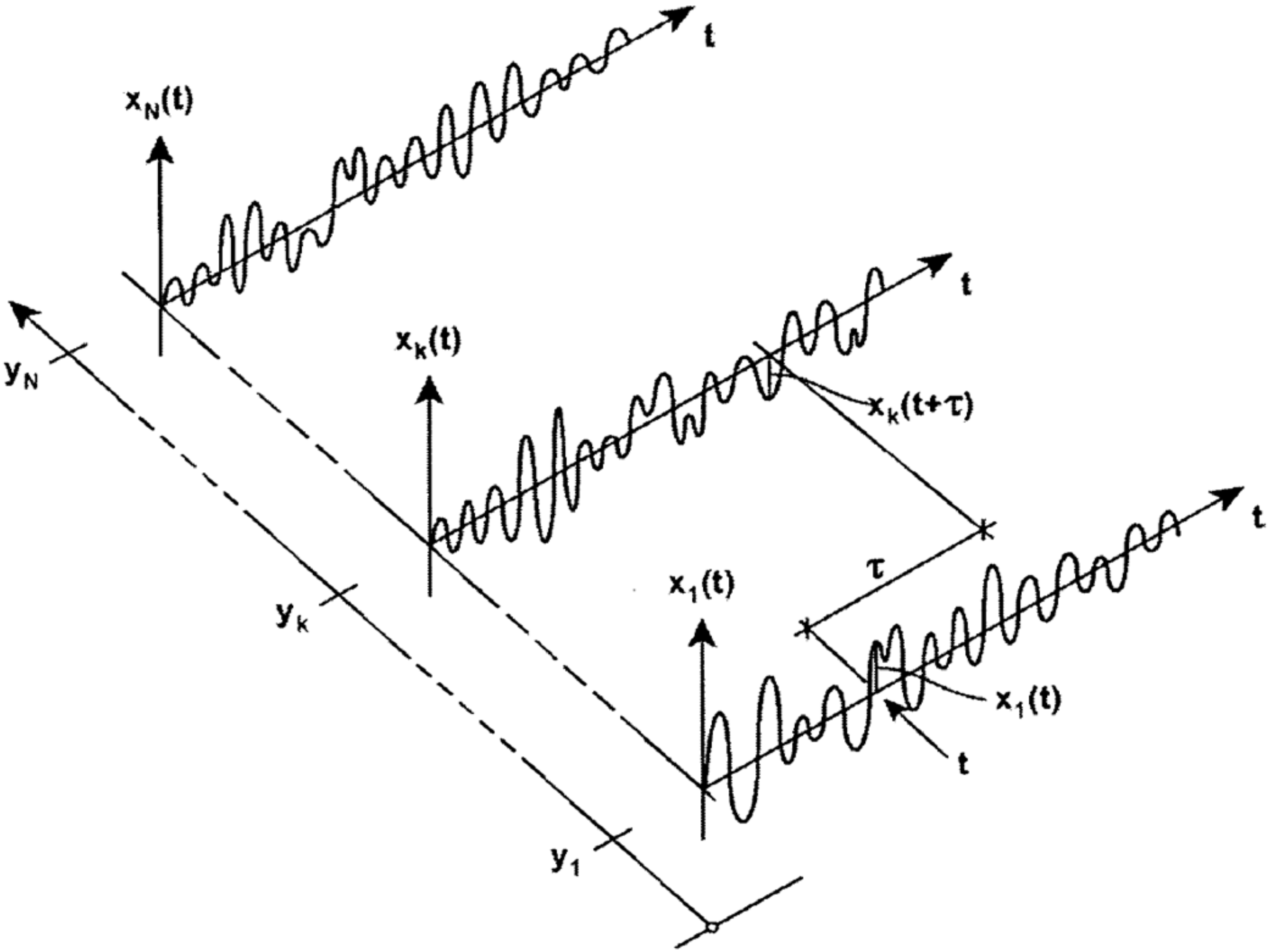
b) *The probability of mean values*

The auto covariance function ②

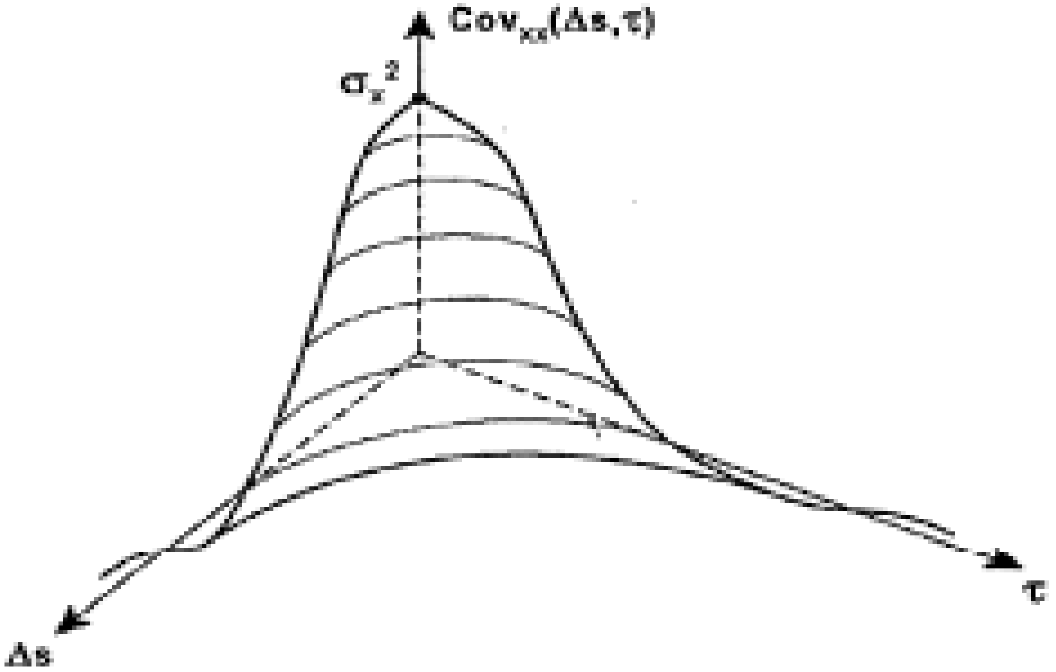


$$Cov_x(\tau = j \cdot \Delta t) = E\left[x(t) \cdot x(t+\tau)\right] = \frac{1}{N-j}\sum_{k=1}^{N-j} x_{k+j} \cdot x_k \qquad (2.20)$$

Cross covariance of time series

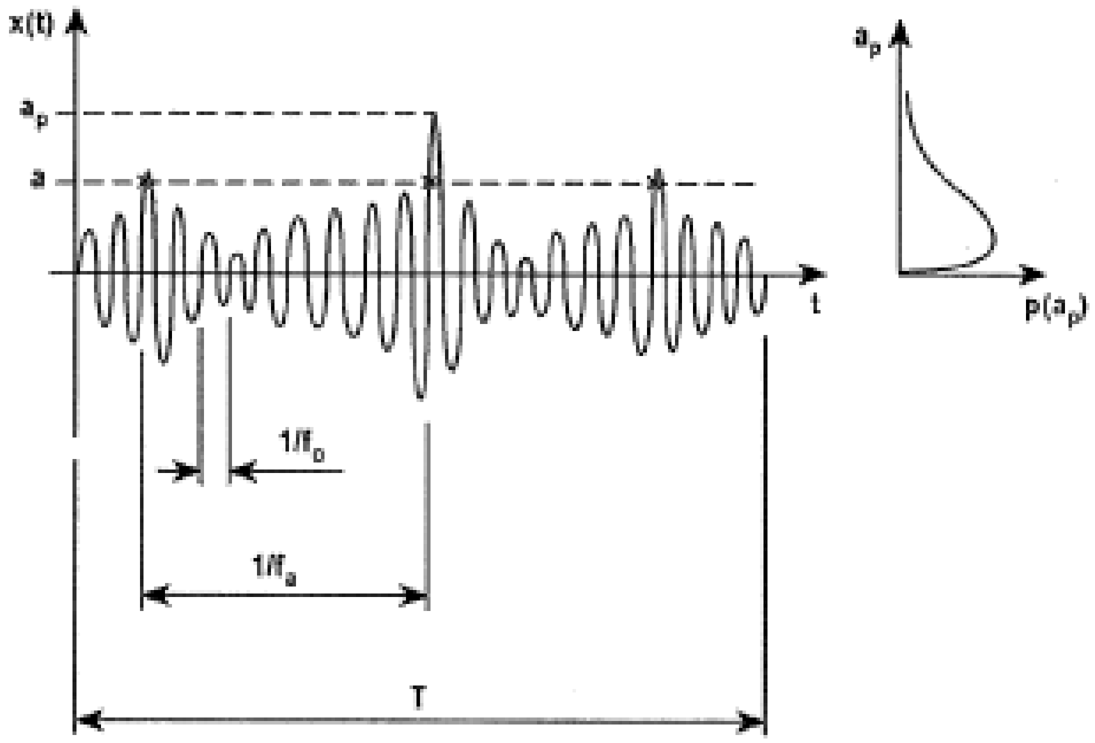


Typical spatial separation and time lag covariance function ②

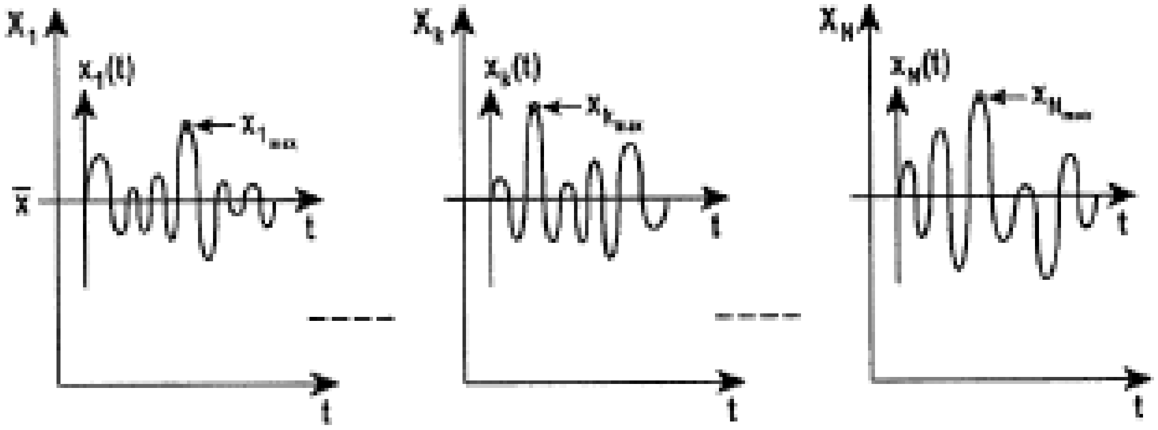


$$Cov_{xx}(\Delta y,\tau)=E\left[x(y,t)\cdot x(y+\Delta y,t+\tau)\right]$$
$$=\lim_{T\to\infty}\frac{1}{T}\int_0^T x(y,t)\cdot x(y+\Delta y,t+\tau)\,dt \qquad (2.24)$$

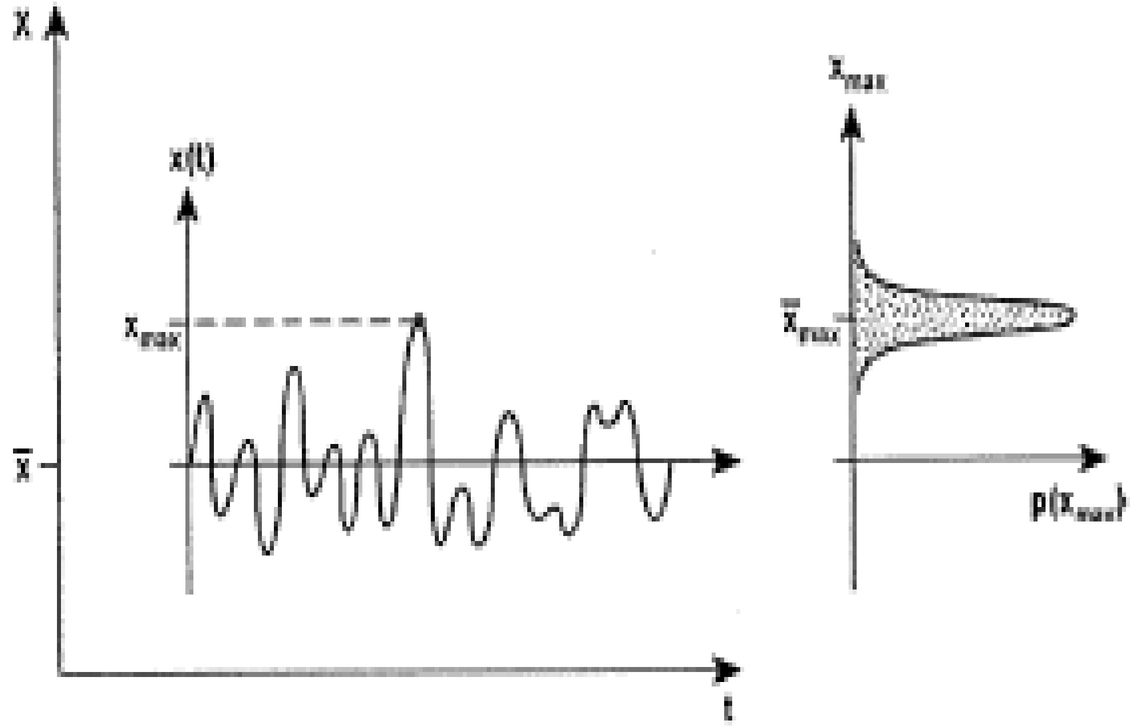
Threshold crossing and peaks



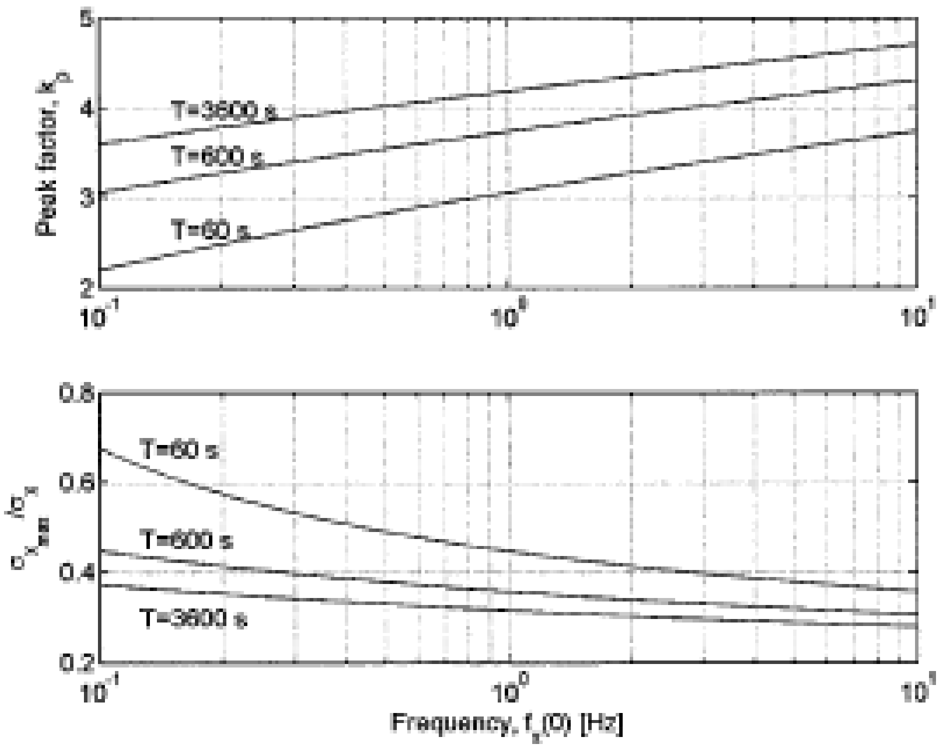
N short term independent realization



The distribution of extremes



Plots of k_p and $\sigma_{x_{max}}/\sigma_x$

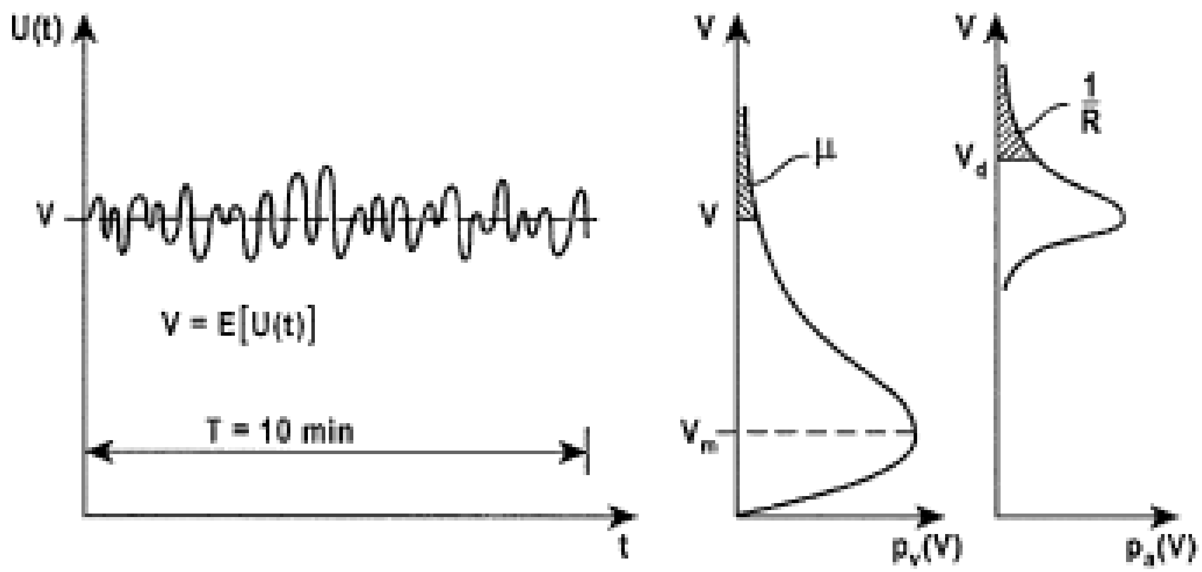


$$X_{max} = \bar{x} + k_p \cdot \sigma_x \tag{2.44}$$

where the peak factor k_p is given by

$$k_p = \sqrt{2 \cdot \ln\left[f_x(0) \cdot T\right]} + \frac{\gamma}{\sqrt{2 \cdot \ln\left[f_x(0) \cdot T\right]}} \tag{2.45}$$

The probability density distribution of mean wind velocity



Aerodynamic admittance

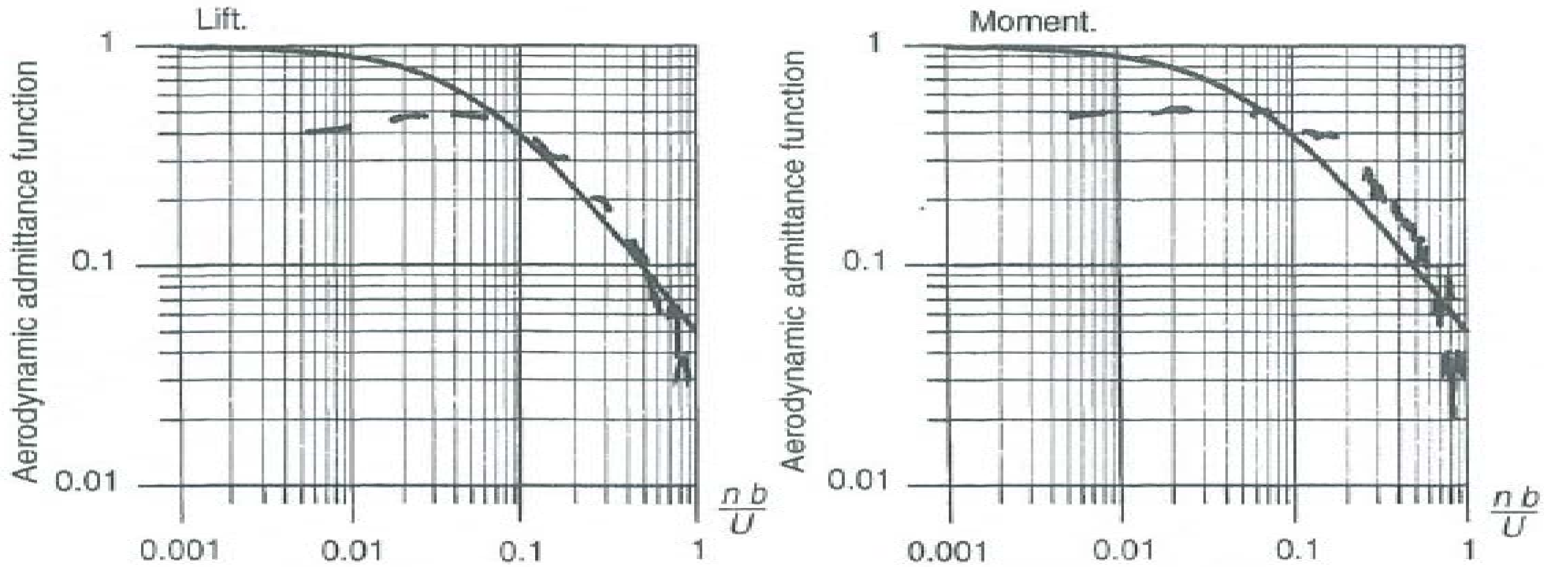


Fig. 8.11 *Aerodynamic admittance functions between the vertical turbulence component and the wind load. The so-called Sears function calculated for thin, symmetrical airfoils is shown for reference as a solid line, see equation (8.3.16). The aerodynamic admittance function shown by the dotted line has been determined using a wind-tunnel test with a streamlined box-girder bridge section (Davenport et al. 1992). (Reproduced by permission of A.A. Balkema, Rotterdam).*

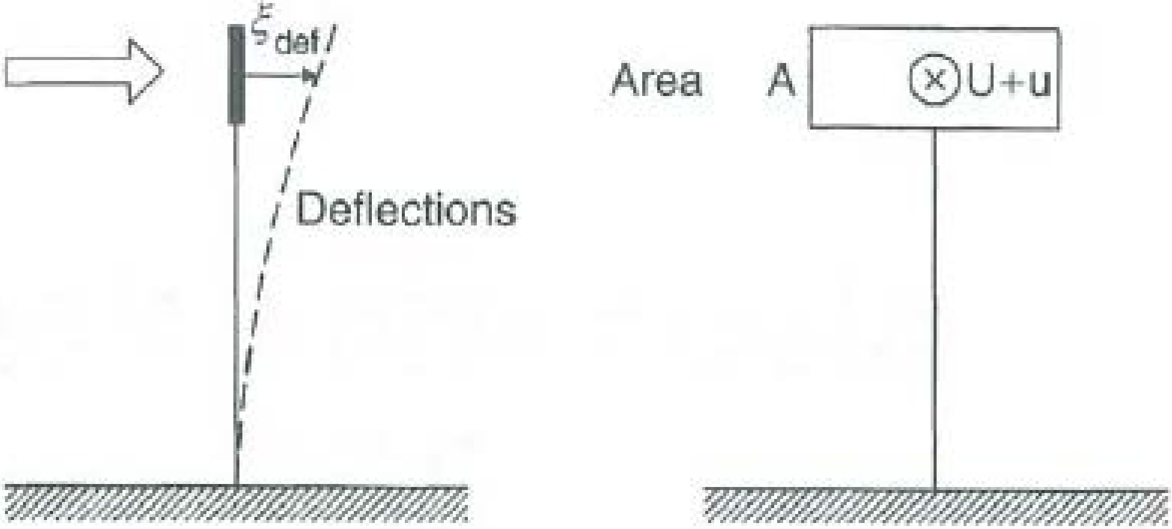
Gust Analysis ③

* The concept of characteristic wind load F_{max}, which was introduced in Section 4.1, is frequently used in wind engineering. The gust factor φ is defined as the ratio between F_{max} and the mean wind load F_q. For an SDOF structure, φ is expressed as

$$\varphi = \frac{F_{max}}{F_q} = 1 + k_p 2 I_u \sqrt{k_b + k_r} \tag{5.1.1}$$

k_p is the peak factor, defined as the ratio between the expected maximum of the fluctuating part of the response and the standard deviation of the response,

Structure with S-D-F-System



Wind Load on Point-Like Structures

It has been assumed that the structure can be modelled as a mass m supported by an elastic spring with stiffness k acting in parallel with a viscous damper with the damping coefficient c_s. The deflection ξ_{def} follows from

$$m\ddot{\xi}_{\mathrm{def}} + c_s\dot{\xi}_{\mathrm{def}} + k\xi_{\mathrm{def}} = F_{\mathrm{tot}} \qquad (5.2.1)$$

where F_{tot} is the along-wind load on the structure.

Calculation of Load

$$(U + u - \dot{\xi}_{\text{def}})^2 = U^2 + 2Uu - 2U\dot{\xi}_{\text{def}} \tag{5.2.3}$$

The total wind load is split into three components: a mean wind load F_q, a fluctuating load F_t caused by the turbulence, and an aerodynamic damping load F_a:

$$F_{\text{tot}} = F_q + F_t - F_a \tag{5.2.4}$$

$$F_q = C_D A \tfrac{1}{2} \rho U^2 \tag{5.2.5}$$

$$F_t = C_D A \rho U u \tag{5.2.6}$$

$$F_a = C_D A \rho U \dot{\xi}_{\text{def}} = c_a \dot{\xi}_{\text{def}} \tag{5.2.7}$$

$$c_a = C_D A \rho U \tag{5.2.8}$$

Total Damping : c

The aerodynamic damping constant c_a is added to the structural damping constant c_s, giving a total damping constant c:

$$c = c_a + c_s \tag{5.2.9}$$

Mean deflection： μ_ξ

The mean deflection μ_ξ is the mean wind load F_q divided by the structural stiffness k,

$$\mu_\xi = \frac{F_q}{k} \tag{5.2.10}$$

and F_q is given by formula (5.2.5).

Structural Vibration

The autospectrum $S_\xi(n)$ for the deflection is determined by

$$S_\xi(n) = |H(n)|^2 S_F(n) \tag{5.2.11}$$

in which $H(n)$ is the the frequency response function for the structure and $S_F(n)$ is the autospectrum for the load,

Variance of deflection : σ_ξ^2

$$\sigma_\xi^2 = \int_0^\infty S_\xi(n)\,dn = \frac{4F_q^2}{k^2}\frac{\sigma_u^2}{U^2}\int_0^\infty k^2|H(n)|^2\frac{S_u(n)}{\sigma_u^2}\,dn \qquad (5.2.12)$$

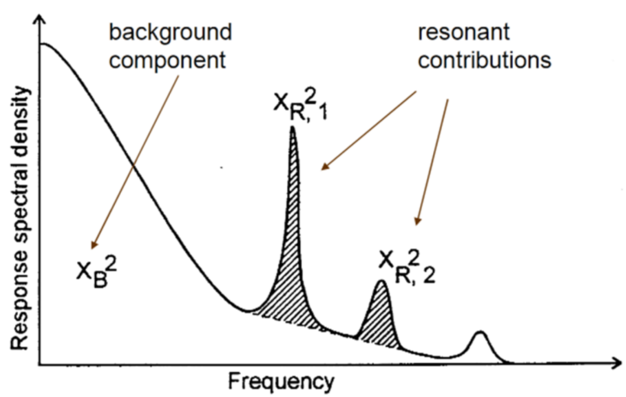
Inserting the turbulence intensity $I_u = \sigma_u/U$ and using (5.2.10) gives

$$\frac{\sigma_\xi}{\mu_\xi} = 2I_u\sqrt{\int_0^\infty k^2|H(n)|^2\frac{S_u(n)}{\sigma_u^2}\,dn} \qquad (5.2.13)$$

Provided that the natural frequency n_e is not very low, it is a good approximation to calculate the integral as the sum $k_b + k_r$ (k_b is the contribution from low frequency turbulence and k_r is the contribution from turbulence in resonance with the structure):

$$k_b = \int_0^\infty k^2 |H(n=0)|^2 \frac{S_u(n)}{\sigma_u^2} dn = 1 \tag{5.2.14}$$

$$k_r = \int_0^\infty k^2 |H(n)|^2 \frac{S_u(n)}{\sigma_u^2} dn = \frac{n_e S_u(n_e)}{\sigma_u^2} \frac{\pi}{4\zeta} \tag{5.2.15}$$



Damping ratio

and finally from (5.2.13)

$$\frac{\sigma_\xi}{\mu_\xi} = 2I_u\sqrt{k_b + k_r} \tag{5.2.16}$$

ζ in (5.2.15) is the damping ratio given by

$$\zeta = \frac{c_a + c_s}{2\sqrt{mk}} \tag{5.2.17}$$

Wind Load on Large Structures

The reduced spatial correlation of wind pressure is important when considering large structures. As shown in Section 4.4, this is done by means of the aerodynamic admittance function. Then factors k_b and k_r are expressed as

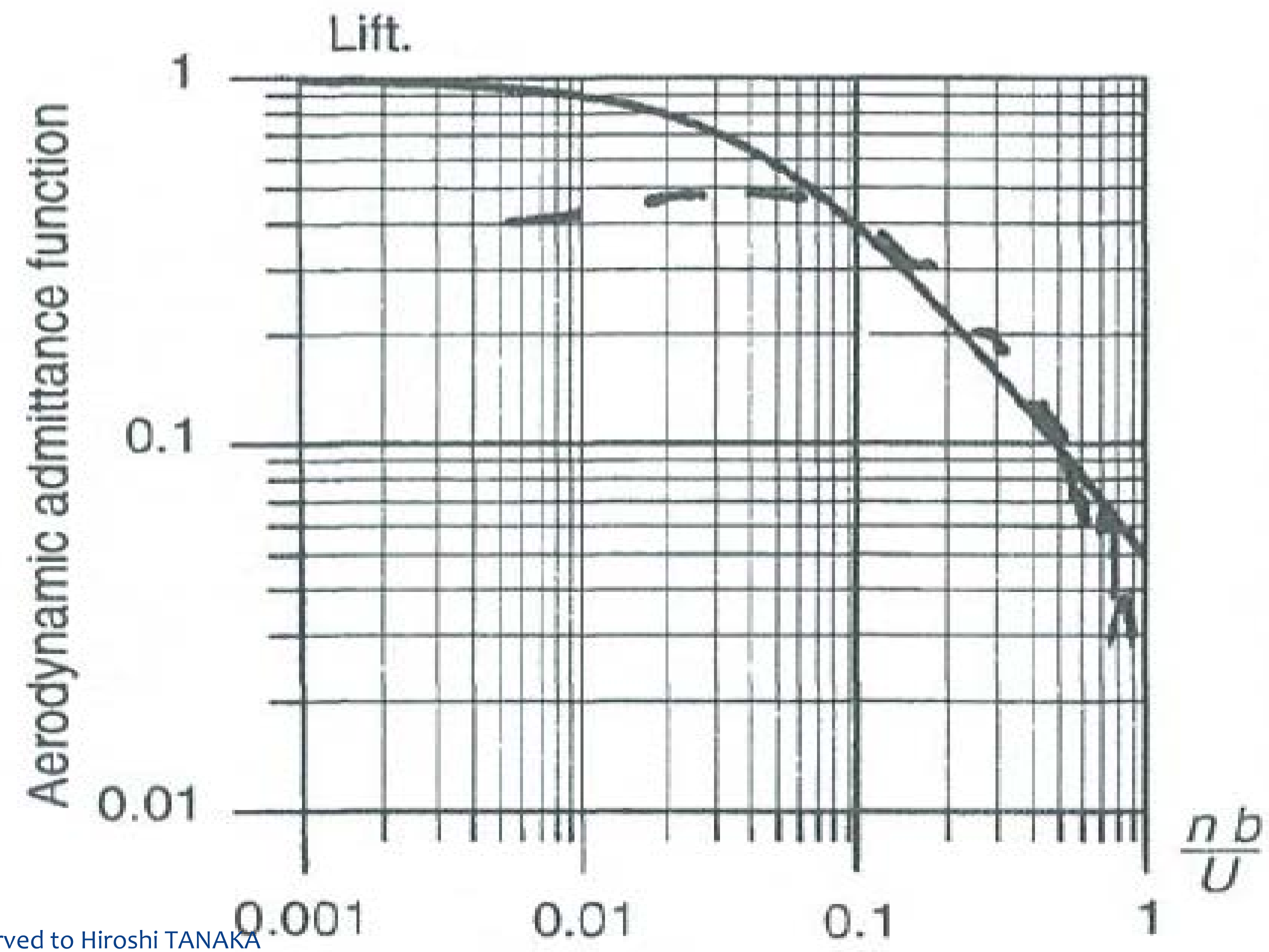
$$k_b = \int_0^\infty \chi^2 \left(\frac{nl}{\bar{U}}\right) \frac{S_u(n)}{\sigma_u^2} dn \tag{5.3.1}$$

$$k_r = \chi^2 \left(\frac{n_e l}{U}\right) \frac{n_e S_u(n_e)}{\sigma_u^2} \frac{\pi}{4\zeta} \tag{5.3.2}$$

in which χ^2 is the aerodynamic admittance function and l is a characteristic length of the wind exposed area. $S_u(n)$ and σ_u are related to the centre of this area.

The aerodynamic admittance function is less than or equal to 1 for all values of its argument, and consequently k_b is also less than or equal to 1.

Aerodynamic admittance
Use as Drag not Lift



Gust response factor

The characteristic deflection ξ_{max} during a certain period of time is expressed as the mean deflection μ_ξ plus the peak factor k_p multiplied by the standard deviation σ_ξ,

$$\xi_{max} = \mu_\xi + k_p \sigma_\xi \tag{5.4.1}$$

The gust factor is

$$\varphi = \frac{\xi_{max}}{\mu_\xi} = 1 + k_p \frac{\sigma_\xi}{\mu_\xi} \tag{5.4.2}$$

and using (5.2.16),

$$\varphi = 1 + k_p 2 I_u \sqrt{k_b + k_r} \tag{5.4.3}$$

The peak factor

The peak factor is discussed in Section 4.6. Here, however, the zero-upcrossing frequency ν should be taken as a weighted average from background and resonance contributions, and the time $T = 600$ s, as a 10-minute mean value is used to determine the mean wind velocity. Then from (4.6.2)

$$k_p = \sqrt{2\ln(\nu T)} + \frac{0.577}{\sqrt{2\ln(\nu T)}} \tag{5.4.4}$$

$$\nu = \sqrt{\frac{n_0^2 k_b + n_e^2 k_r}{k_b + k_r}} \tag{5.4.5}$$

Representative frequency (Hz)

where n_e is the resonant frequency (Hz) for the along-wind vibrations of the structure and n_0 is the representative frequency (Hz) of the gust loading on rigid structures. The frequency n_0 is determined as

$$n_0 = \sqrt{\frac{\int_0^\infty n^2 \chi^2 \left(\frac{nl}{U}\right) S_u(n)\, dn}{\int_0^\infty \chi^2 \left(\frac{nl}{U}\right) S_u(n)\, dn}} \tag{5.4.6}$$

Von Karman power-spectral density

$$R_N(z,n) = \frac{4 f_L}{(1 + 70.8 f_L^2)^{5/6}} \tag{3.5.18}$$

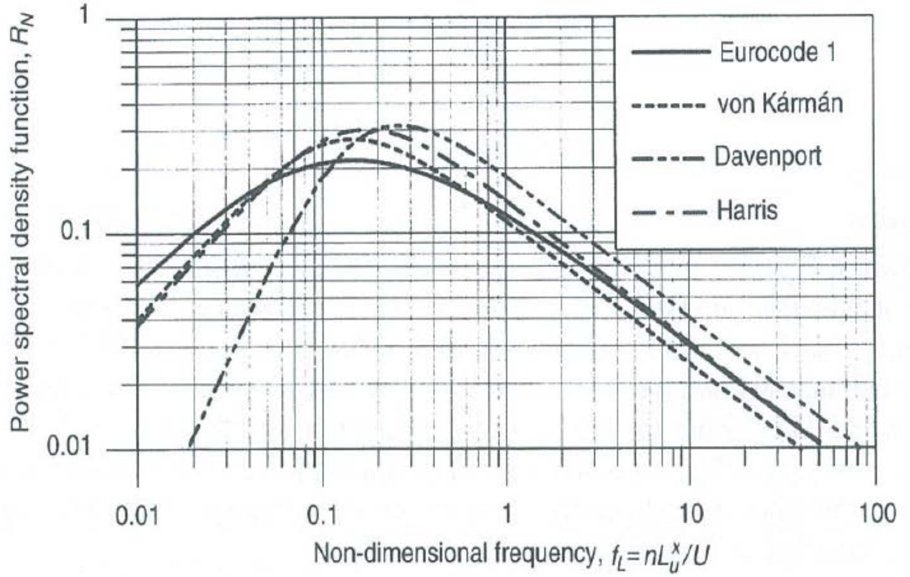


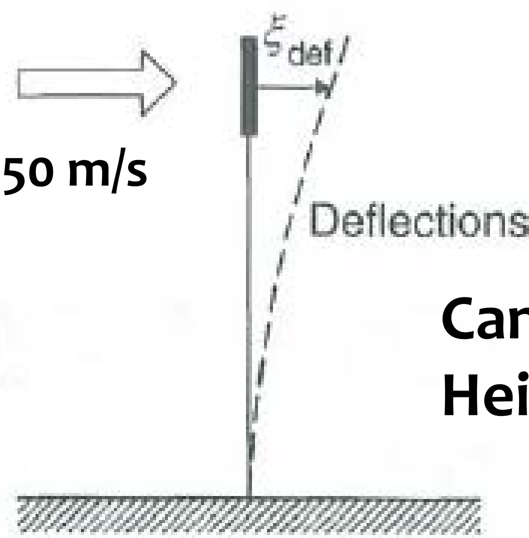
Fig. 3.21 *Power-spectral density functions for the longitudinal turbulence component. The integral length scale L_u^x has been assumed to be 180 m when plotting the spectra suggested by Davenport (3.5.20) and Harris (3.5.21). For $f_L >$ approximately 0.2, the Davenport spectrum gives the largest spectral values. Compared to Eurocode 1 (3.5.17) the von Kármán spectrum (3.5.18) gives slightly lower spectral values for $f_L >$ approximately 1, see also Section 3.5.5.*

Example of Calculation

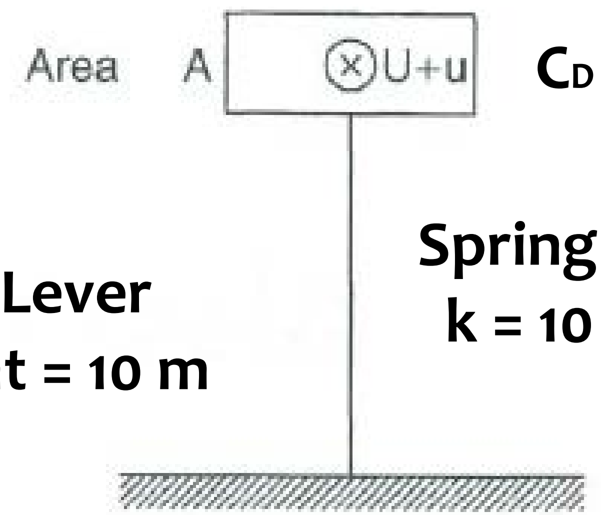
* We will try to do gust analysis following input
* Wind speed :   50m/s
* Turbulent intensity Iu = 0.1 (10%) ;Turbulent Scale   Lu = 100m
* Area is 1m x 1m   =   1m^2
* Weight   **10t**

Wind

50 m/s



Canti-Lever
Height = 10 m



C_D = 1.5

Spring constant :
k = 10 t/m

Wind Loads & Mean Deflection

- From Eq. (5.2.5)

  Fq = 0.5 x C_D x A x ρ x U^2

  = 0.5 x 1.5 x 1(m^2) x 0.123(kgs^2/m^4)x 50(m/s)^2

  = 230 kg = **0.23 t** by Mean wind force

Mean deflection by Eq. (5.2.10)

$$\mu_\xi = Fq \;/\; k = 0.23 \;/\; 10 = \mathbf{0.023\ m}$$

Eigen frequency （f :ne)

* 2 π f = √ k /m

= √ （10x10ˆ3 (kg/m)) / 10x10ˆ3 (kg)

/9.8 (m/s^2)

= √9.8 (1/s^2)

f = 0.5 (1/s)

= 0.5 (Hz)

= ne

Damping Ratio (ζ)

- ζ is the sum of ζa + ζs
- ζa : damping by wind
- ζs : structural damping = 0.01 (1 %)

- From Eq.(5.2.8) Ca = C_D x A x ρ x U

  = 1.5 x 1 x 0.123 x 50

  = 9.225

  ζa = Ca / (2√mk)

  = 0.00144

  ζ = ζa + ζs = 0.01144

ζ in (5.2.15) is the damping ratio given by

$$\zeta = \frac{c_a + c_s}{2\sqrt{mk}} \tag{5.2.17}$$

Von Karman power-spectral density: R_N

* **f_L = n_e x L_u/ U**

  $= 0.5(\mathrm{Hz}) \times 100\ (\mathrm{m}) / 50(\mathrm{m/s})$

  $= 1$

$R_N(z, n) = 4\ f_L / (1 + 70.8\ f_L{}^{\wedge}2)^{5/6}$

$= 4 / (1 + 70.8)^{5/6}$

$= 0.114$

* Turbulent intensity $I_u = 0.1$ (10%)

Then $Iu = \sigma u \;/\; U =$ 5(m/s)/50(m/s)

$\sigma u = 5$ (m/s)

Kb & Kr

- $Kb = 1$ (5.2.14)
- $Kr = (ne \times Su(ne) \times \pi) / (\sigma u \,\hat{}\, \mathbf{2} \;\; \mathbf{x} \;\; \mathbf{4\zeta})$

$= (0.5 \times 0.114 \times 3.14)/(5 \,\hat{}\, \mathbf{2} \times 4 \times 0.01144)$

$= 0.156$

$\sigma\xi \;/\; \mu\xi \;=\; 2\, lu\, \surd\; (kb + kr)$ (5.2.16)

$= 2 \times 0.1 \surd\; (1 + 0.156)$

$= 0.22$

$\sigma\xi = \mu\xi \times 0.22 = 0.023 \times 0.22 = 0.005$

Aerodynamic admittance (A-Ad)

* This example is small strucrure
* Then A-Ad is 1

Representative Frequency (n_0)

$$n_0 = \sqrt{\frac{\int_0^\infty n^2 \chi^2 \left(\frac{nl}{U}\right) S_u(n)\,dn}{\int_0^\infty \chi^2 \left(\frac{nl}{U}\right) S_u(n)\,dn}} \qquad (5.4.6)$$

n_0 is difficult to calculate then approximate values are possible by the following method:
We use relationship of (2.34) in text book ②
n_0 is identical to the following $f_x(0)$

$$f_x(0) = \frac{1}{2\pi} \cdot \frac{\sigma_{\dot{x}}}{\sigma_x} \qquad (2.34)$$

Approximation

$\sigma_{\dot{x}}$ 2πfe) x σ_x

Using the following equation:

$$f_x(0) = \frac{1}{2\pi} \cdot \frac{\sigma_{\dot{x}}}{\sigma_x} \qquad (2.34)$$

$f\,x\,(0) = f\,e = 0.5\ Hz$

$$\nu = \sqrt{\frac{n_0^2 k_b + n_e^2 k_r}{k_b + k_r}} \quad (5.4.5)$$

* *n0* = *ne* = 0.5 Hz
* *Kb* = *1* *Kr* = 0.156
* Input them in Eq.(5.4.5)

* ν = 0. 50

$$k_p = \sqrt{2\ln(\nu T)} + \frac{0.577}{\sqrt{2\ln(\nu T)}} \tag{5.4.4}$$

- T = 600 sec
- ν = 0. 50
- Input them in Eq.(5.4.4)

- kp = 3.55

Maximum Response　ξ max

* $\xi max = \mu_{\xi} + kp \cdot \sigma_{\xi}$

$$= 0.023 + 3.55 \times 0.005$$

$$= 0.041$$

Gust Factor ψ

* $\psi = \xi_{max} / \mu_\xi$

  $= 0.041 / 0.023$

  $= 1.78$

Generally speaking; $\psi = 1.7 \sim 1.9$

Conclusions

1) Gust response analysis is important for flexible structures, especially, suspension bridges and cable-stayed bridges.

2) Large scale structures (i.e. Multi-Degree-Of-Freedom-System), FEM program is necessary.

3) It is important to make wind tunnel tests in turbulent flow. In this case, gust response analysis is necessary to confirm the results of the wind tunnel tests.

THANK YOU !