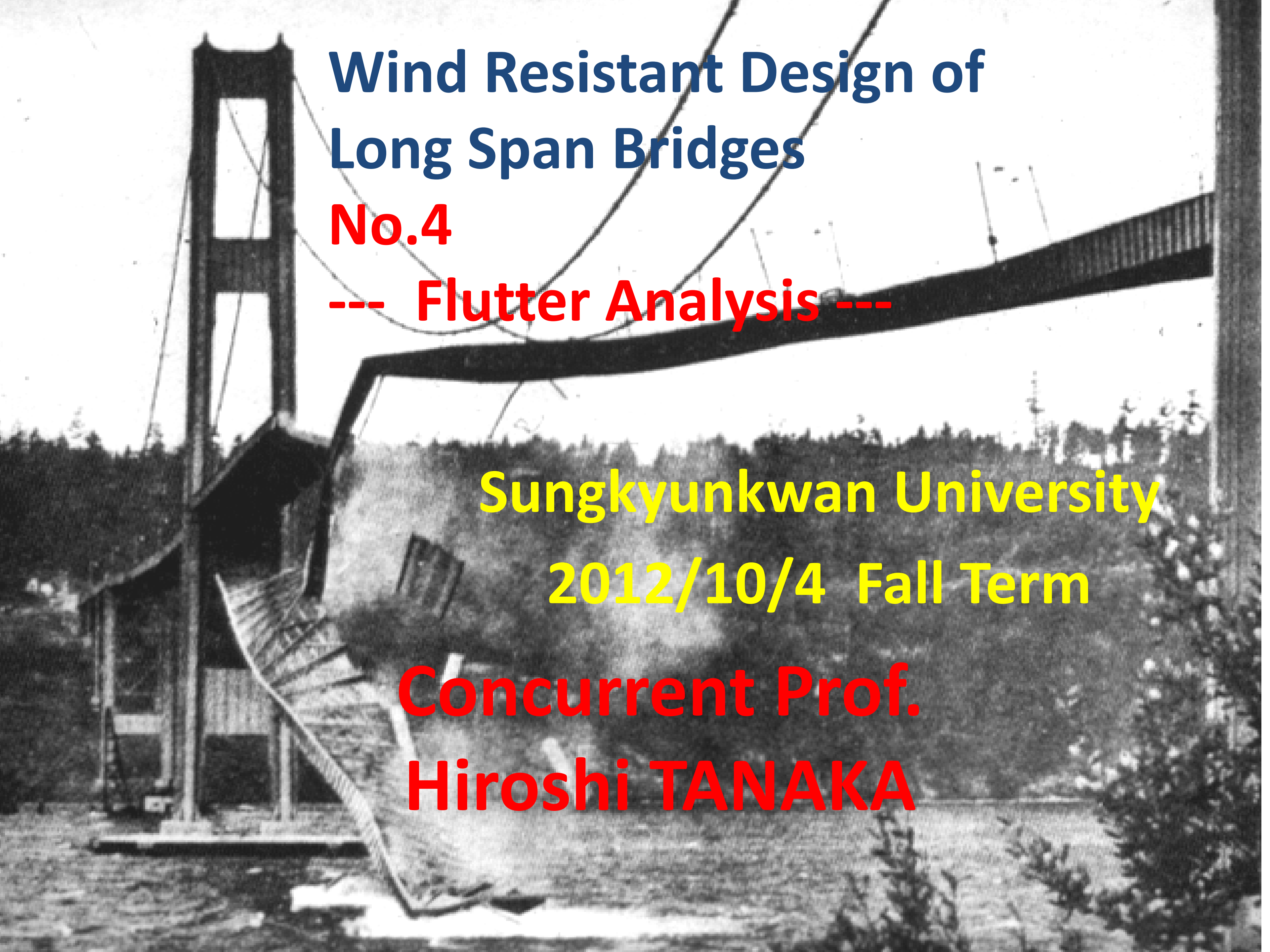

Wind Resistant Design of Long Span Bridges
No.4
--- Flutter Analysis ---

Sungkyunkwan University

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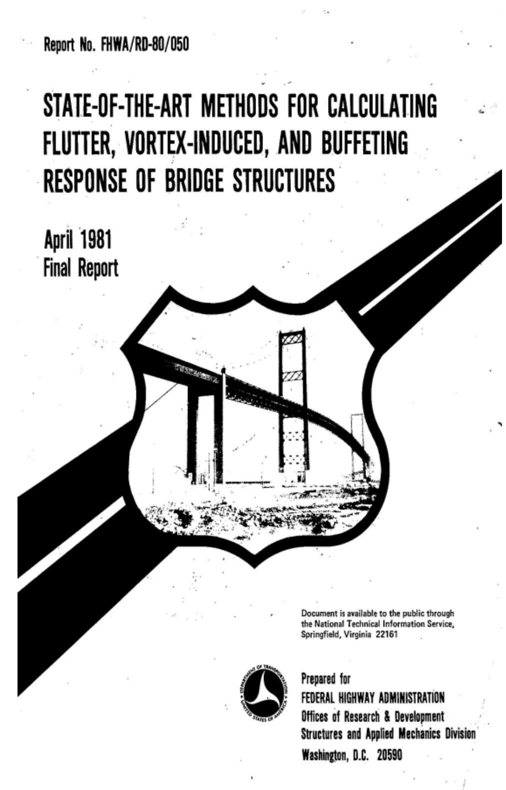
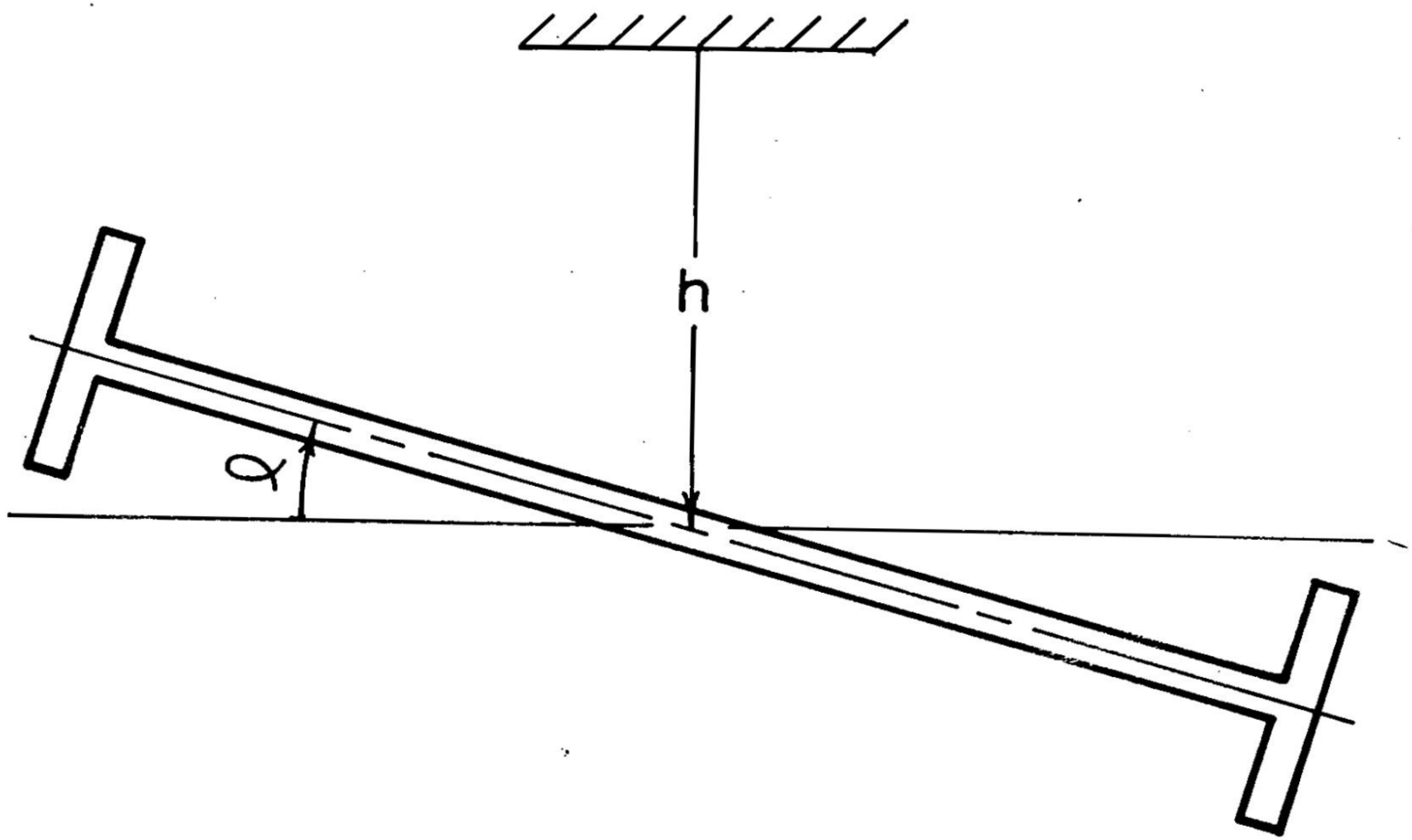
Method

- Flutter analysis is based upon Prof. Scanlan's Method.
  1) His method is simple and easy by hands.
  2) Preliminary course is assumed.
  3) Suspension bridge is main target.

Reference
R.H. Scanlan : State-of-the-Art Methods for Calculating Flutter, Vortex-Induced, and Buffeting Response of Bridge Structures
Report No. FHWA/RD-80/050
April 1981 Final Report



Analytics of the Flutter Problem



DEGREES OF FREEDOM FOR DECK SECTION

FIGURE 3.1

The 2-DOF Problem

Consider the bridge deck section as pictured in Fig. 3.1. Let h represent the vertical deflection of the local c.g. of the section and α the rotation coordinate (angle) about that c.g.. Let m represent the mass per unit span and I, the mass moment of inertial about the c.g., per unit span. Then (neglecting lateral motion as unimportant to flutter) the two sectional equations of motion are

(3.1a) $$M[\ddot{h} + 2\zeta_h \omega_h \dot{h} + \omega_h^2 h] = L_h$$

(3.1b) $$I[\ddot{\alpha} + 2\zeta_\alpha \omega_\alpha \dot{\alpha} + \omega_\alpha^2 \alpha] = M_\alpha$$

where ζ_h, ζ_α are the damping ratios-to-critical and ω_h, ω_α are the natural circular frequencies, respectively in h- and α- motions, and L_h, M_α are the aerodynamic force and moment per unit span acting on the section.

its c.g.), the corresponding equations become

$$m[\ddot{h} + a\ddot{\alpha} + 2\,\zeta_h \omega_h \dot{h} + \omega_h^2 h] = L_h \tag{3.2a}$$

$$I[\ddot{\alpha} + \frac{a}{r_g^2}\ddot{h} + 2\,\zeta_\alpha \omega_\alpha \dot{\alpha} + \omega_\alpha^2 \alpha] = M_\alpha \tag{3.2b}$$

where $m\,a$ is the mass unbalance of the section about its c.g., and r_g is its radius of gyration about the same point.

The aerodynamic force and moment at the c.g. are of the linear, self-excited type and are basically given by

$$\frac{L_h}{m} = H_1\dot{h} + H_2\dot{\alpha} + H_3\alpha \tag{3.3a}$$

$$\frac{M_\alpha}{I} = A_1\dot{h} + A_2\dot{\alpha} + A_3\alpha \tag{3.3b}$$

where the coefficients H_i, A_i $(i=1,2,3)$ are aerodynamic in origin and must be determined experimentally for the particular shape of deck in question.

The coefficients H_1, pertaining to $\dot{h}$, and A_2 and A_3, pertaining to $\dot{\alpha}$ and α, are the <u>direct</u> coefficients, and the others (H_2, H_3, and A_1) are the <u>coupling</u> coefficients. In many instances, the direct coefficients prove to be the more important.

$$L_h = \frac{1}{2} \rho U^2 (2B)\left[K H_1^* \frac{\dot{h}}{U} + K H_2^* \frac{B\dot{\alpha}}{U} + K^2 H_3^* \alpha\right] \tag{3.3c}$$

$$M_\alpha = \frac{1}{2} \rho U^2 (2B^2)\left[K A_1^* \frac{\dot{h}}{U} + K A_2^* \frac{B\dot{\alpha}}{U} + K^2 A_3^* \alpha\right] \tag{3.3d}$$

where

ρ = air density $\begin{cases} \rho = 0.002378 \text{ slugs/ft}^3 \\ \rho = 1.228 \text{ kg/m}^3 = 1.228 \times 10^{-3} \text{ gm/cm}^3 \end{cases}$

U = cross wind velocity

$K = \frac{B\omega}{U}$

B = deck width

ω = circular frequency of flutter oscillation and the nondimensional aerodynamic coefficients H_i^* and A_i^* bear the following relation to H_i and A_i :

$$(3.4)\qquad \begin{cases} H_1^* = \dfrac{m\,H_1}{\rho\,B^2\omega}\,; & A_1^* = \dfrac{I\,A_1}{\rho\,B^3\omega} \\[2ex] H_2^* = \dfrac{m\,H_2}{\rho\,B^3\omega}\,; & A_2^* = \dfrac{I\,A_2}{\rho\,B^4\omega} \\[2ex] H_3^* = \dfrac{m\,H_3}{\rho\,B^3\omega^2}\,; & A_3^* = \dfrac{I\,A_3}{\rho\,B^4\omega^2} \end{cases}$$

Some examples of the values experimentally obtained for H_i^* and A_i^* are illustrated in Figs. 3.2 and 3.3 for decks whose sectional form is sketched on the figures. The plots are given as functions of $\dfrac{U}{NB} = \dfrac{2\pi}{K}$.

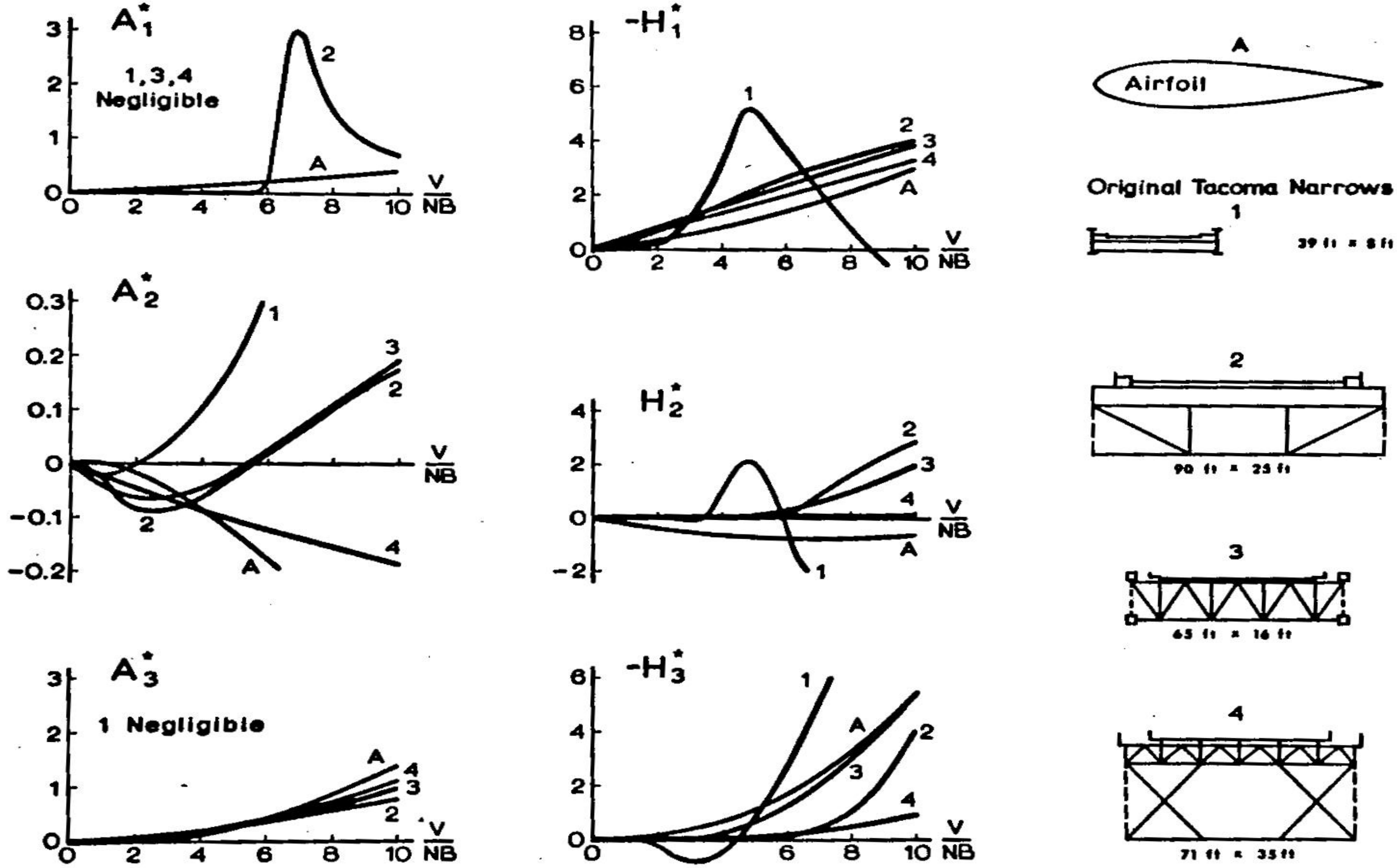


FIGURE 3.2

AERODYNAMIC COEFFICIENTS
(Flutter Derivatives)

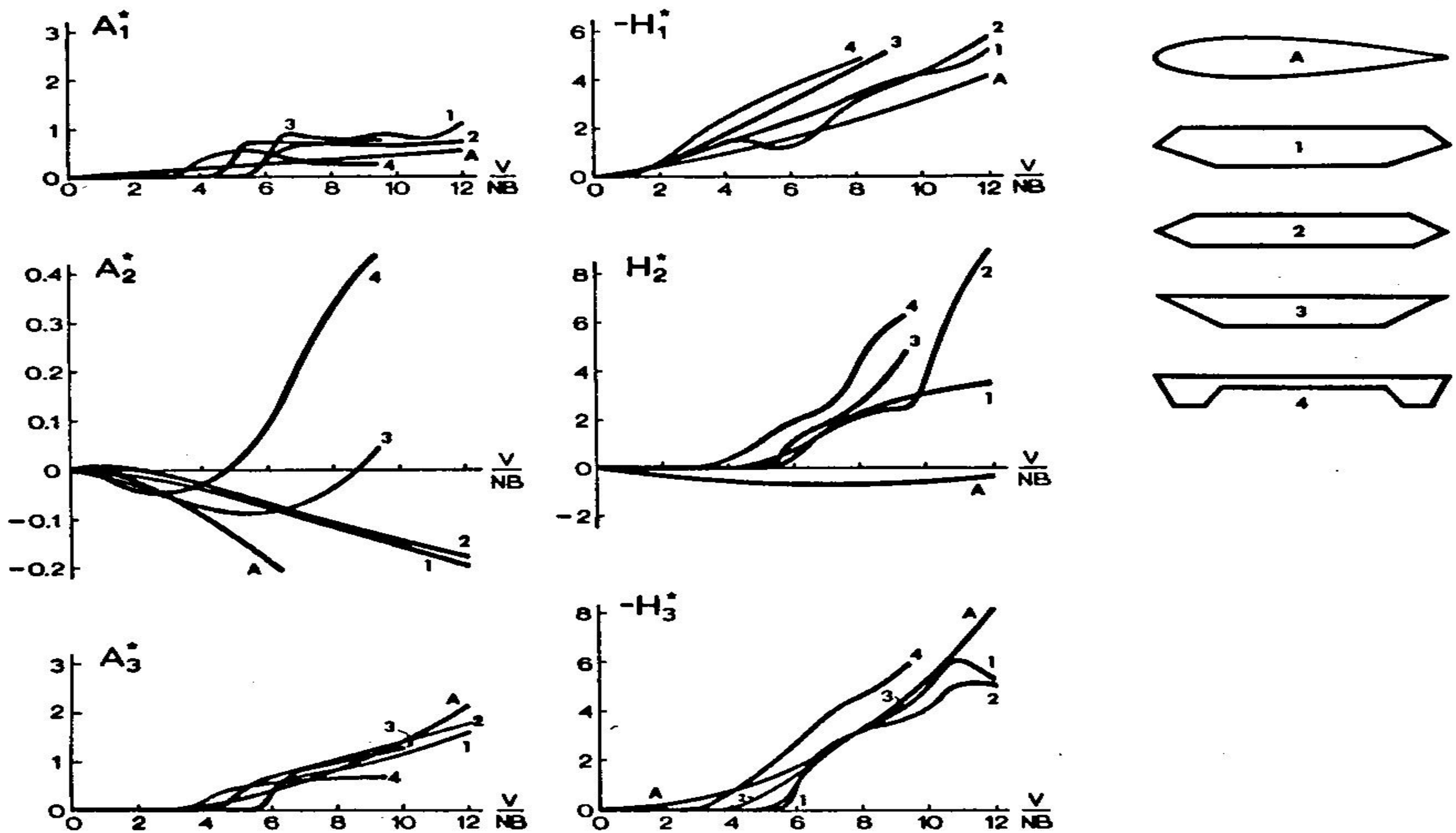


FIGURE 3.3

AERODYNAMIC COEFFICIENTS

Flutter Derivatives (1)

It should be noted that the products KH_1^*, KA_1^*, etc., play the roles of flutter derivatives. Consider the term $H_1\dot{h}$, for example. This has the dimension of a force per unit span length. If written in classic aerodynamic lift force form, it would have the appearance

$$\frac{1}{2}\rho U^2(2B)\ KH_1^*\frac{\dot{h}}{U} = m H_1\dot{h} = \frac{1}{2}\rho U^2(2B)C_L \cong \frac{1}{2}\rho U^2(2B)\frac{dC_L}{d\bar{\alpha}}\frac{\dot{h}}{U}$$

where $\bar{\alpha} = \frac{\dot{h}}{U}$ is an "effective" angle of attack.

Thus,

$$K H_1^* = \frac{dC_L}{d\bar{\alpha}}$$

where $dC_L/d\bar{\alpha}$ is the derivative of a lift coefficient C_L with respect to angle of attack.

Flutter Derivatives (2)

It is emphasized again that the values of all the aerodynamic derivatives must be experimentally obtained, and they evolve as functions of reduced velocity U/NB. Of particular interest is the manner in which the coefficient A_2^* evolves with U/NB. This coefficient is proportional to <u>torsional aerodynamic damping</u>, and it plays a central role in many cases of bridge flutter susceptibility, since it often <u>changes sign</u> (from stable to unstable) with increasing U/NB in certain cases.

The 3-D Problem

the spanwise position x and time t. It will be assumed in this case, that bending and torsion modes of the bridge are independent (uncoupled) from each other. Let $h(x)$ be the modal deflection form in the lowest bending mode and $\alpha(x)$ be the modal deflection form in the lowest torsional mode. Then,

$$h(x,t) = h(x)\, p(t)$$

$$\alpha(x,t) = \alpha(x)\, q(t)$$

where p and q are now the generalized coordinates. The equations of motion (3.1) for the balanced deck then convert immediately to

$$(3.5a) \quad M_1[\ddot{p} + 2\,\zeta_h\omega_h\,\dot{p} + \omega_h^2 p] = \frac{1}{2}\,\rho U^2(2B)[KC_{11}H_1^*\,\frac{\dot{p}}{U} + KC_{12}H_2^*\,\frac{B\dot{q}}{U} + K^2C_{12}H_3^*\,q]$$

$$(3.5b) \quad I_1[\ddot{q} + 2\,\zeta_\alpha\omega_\alpha\,\dot{q} + \omega_\alpha^2 q] = \frac{1}{2}\,\rho U^2(2B^2)[KC_{12}A_1^*\,\frac{\dot{p}}{U} + KC_{22}A_2\,\frac{B\dot{q}}{U} + K^2C_{22}A_3^*\,q]$$

where

$$(3.6a) \qquad M_1 = \int_{span} m(x)\,h^2(x)dx$$

$$(3.6b) \qquad I_1 = \int_{span} I(x)\,\alpha^2(x)dx$$

$$(3.6c) \qquad C_{11} = \int_{span} h^2(x)dx$$

$$(3.6d) \qquad C_{12} = \int_{span} h(x)\,\alpha(x)dx$$

$$(3.6e) \qquad C_{22} = \int_{span} \alpha^2(x)dx$$

are generalized mass, moment of inertia, and modal factors, respectively. Generalizations of Eqs. (3.2) for the unbalanced deck are direct but will be omitted here. Note that $\int_{span}$ represents an integral over the entire side- and main-spans of the bridge (i.e. over whatever constitutes the mode in question).

Solution of the Flutter Equations

① *Pure Torsional Flutter*

If the torsional coefficient A_2^* reverses sign with increasing $\frac{U}{NB}$ (see, for example, Fig. 3.4), flutter instability is indicated. This is, in fact, a very common case in practice. To find the wind velocity at flutter ("critical flutter velocity"), proceed as follows.

(In effect, this states that the critical flutter value of A_2^* is equal to the "scaled" value of mechanical damping ζ_α in the α-mode by a factor equal to twice the ratio of the generalized mechanical inertia to the generalized aerodynamic inertia.)

The corresponding value $(U/NB)_c$, is the critical one, and the critical wind velocity is

$$U_c = NB\left(\frac{U}{NB}\right)_c \tag{3.2}$$

<u>Note</u>: If A_2^* <u>never changes sign,</u> the critical flutter velocity <u>may</u> be high, but in any case, it depends upon solution of the two-degree-of-freedom problem (in both p and q). This is treated below.

In Eq. (3.5b), consider only the damping terms (proportional to $\dot{q}$).

Mechanical damping is just balanced out by aerodynamic damping if:

$$I_i(2\zeta_\alpha\omega_\alpha)\dot{q} = \rho U^2B^2 K C_{22}A_2^* \frac{B\dot{q}}{U} \tag{3.19}$$

Assuming (as is usually the case), that flutter occurs practically at the same frequency as the first mode, let $\omega \cong \omega_\alpha$. Then the condition (3.19) yields the critical value of A_2^* as

$$A_2^* \cong \frac{2 I_1\zeta_\alpha}{\rho B^4 C_{22}} \tag{3.20}$$

② *The 2D-O-F Case for Straight Decks*

This case is more rare and likely to produce flutter only if the lowest bending and torsion frequencies of the bridge are "near" each other. Further, it will not be the important case for a design unless the plot of the coefficient A_2^* does <u>not</u> reverse sign.

When the conditions for two-degree flutter are satisfied (as, occasionally, for very streamlined bridges), the flutter that can occur is said to be of the "classical" airfoil, or "coupled" type. This is a flutter in which interaction of aerodynamic stiffness terms, rather than damping terms, is the principal mechanism.

The solution proceeds by assuming a sinusoidal response jointly in p and q and setting the determinant of the two-equation system (3.5) equal to zero. Details are given in Ref. [3.2]. Letting

$$X = \omega/\omega_h \tag{3.22}$$

where ω is the circular flutter frequency, the following two simultaneous equations, with X as unknown, are obtained:

(3.23a) $$a_4X^4 + a_3X^3 + a_2X^2 + a_1X + a_o = 0$$

(3.23b) $$b_3X^3 + b_2X^2 + b_1X + b_o = 0$$

where the coefficients a_i and b_i are the following constants or functions of K:

(3.24a) $$a_0 = \left(\frac{\omega_\alpha}{\omega_h}\right)^2$$

(3.24b) $$a_1 = 0$$

(3.24c) $$a_2 = \frac{\omega_\alpha^2}{\omega_h^2} - 4\zeta_h\zeta_\alpha \frac{\omega_\alpha}{\omega_h} - 1 - \frac{\rho B^4}{I_1} C_{22}A_3^*$$

(3.24d) $$a_3 = 2\,\zeta_\alpha \frac{\omega_\alpha}{\omega_h} \frac{\rho B^2}{M_1} C_{11}H_1^* + 2\zeta_h \frac{\rho B^4}{I_1} C_{22}A_2^*$$

(3.24e) $$a_4 = 1 + \frac{\rho B^4}{I_1} C_{22}A_3^* + \frac{\rho B^2}{M_1}\frac{\rho B^4}{M_1}(C_{12}^2\, A_1^*H_2^* - C_{11}C_{22}\, A_2^*H_1^*)$$

$$b_0 = 2\,\zeta_h\left(\frac{\omega_\alpha}{\omega_h}\right)^2 + 2\,\zeta_\alpha\,\frac{\omega_\alpha}{\omega_h} \tag{3.25a}$$

$$b_1 = -\frac{\rho B^2}{M_1}\,C_{11}H_1^*\,\frac{\omega_\alpha^2}{\omega_h^2} - \frac{\rho B^4}{I_1}\,C_{22}A_2^* \tag{3.25b}$$

$$b_2 = -2\,\zeta_\alpha\,\frac{\omega_\alpha}{\omega_h} - 2\,\zeta_h - 2\,\zeta_h\,\frac{\rho B^4}{I_1}\,C_{22}A_3^* \tag{3.25c}$$

$$b_3 = \frac{\rho B^4}{I_1}\,C_{22}A_2^* + \frac{\rho B^2}{M_1}\,C_{11}H_1^*\,\frac{\rho B^2}{M_1}\,\frac{\rho B^4}{I_1}\left(C_{11}C_{22}\,H_1^*A_3^* - C_{12}^2A_1^*H_3^*\right) \tag{3.25d}$$

+

The solution method is as follows. A value of K is chosen and all the coefficients H_i^* and A_i^* are evaluated for that K. Then a_i and b_i in (3.12) and (3.13) are evaluated. These constants are then used in Eqs. (3.11). Eqs. (3.11) are then solved for the values of X corresponding to the K chosen. The above process is repeated for a series of values of K.

Plots of the solutions X of (3.23a) and (3.23b) are made vs. the K values used. Where the plot of solutions X for (3.23a) crosses the plot of solutions X from (3.23b), the critical flutter condition X_c exists. The corresponding K value is

Ref.[3.2]（1）

Because of its interest in applications, a useful variant on the solution outlined above is now presented in somewhat more detail. Let

$$s=\frac{Ut}{B}$$

be a nondimensional time (or distance). Noting that

$$(\dot{\ })=\frac{d(\ )}{dt}=\frac{d(\ )}{ds}\frac{ds}{dt}=(\ )'\frac{U}{B}$$

Equations 6.5.2 and 6.5.4 can be reduced to

$$\frac{h''}{B}+2\zeta_h K_h\frac{h'}{B}+K_h^2\frac{h}{B}=\frac{\rho B^2}{m}\left[KH_1^*\frac{h'}{B}+KH_2^*\alpha'+K^2H_3^*\alpha\right] \tag{6.5.7a}$$

$$\alpha''+2\zeta_\alpha K_\alpha\alpha'+K_\alpha^2\alpha=\frac{\rho B^4}{I}\left[KA_1^*\frac{h'}{B}+KA_2^*\alpha'+K^2A_3^*\alpha\right] \tag{6.5.7b}$$

where $K_h=B\omega_h/U$, $K_\alpha=B\omega_\alpha/U$.

- Remarks: Eq.(6.5.7a) &（6.5.7b）here are Eq.(3.1a)&(3.2b) .

Ref.[3.2]（2）

Posing now the solution forms

$$\frac{h}{B}=\frac{h_0}{B}e^{i\omega t}=\frac{h_0}{B}e^{iKs}$$

$$\alpha=\tilde{\alpha}_0 e^{i(\omega t+\phi)}=\alpha_0 e^{i\omega t}=\alpha_0 e^{iKs}$$

Equations 6.5.7 take the form

$$\left[-K^2+2i\zeta_h K_h K+K_h^2-\frac{\rho B^2}{m}iK^2H_1^*\right]\frac{h_0}{B}-\left[\frac{\rho B^2}{m}iK^2H_2^*+\frac{\rho B^2}{m}K^2H_3^*\right]\alpha_0=0 \qquad (6.5.8a)$$

$$\left[-\frac{\rho B^4}{I}iK^2A_1^*\right]\frac{h_0}{B}+\left[-K^2+2i\zeta_\alpha KK_\alpha+K_\alpha^2-\frac{\rho B^4}{I}iK^2A_2^*-\frac{\rho B^4}{I}K^2A_3^*\right]\alpha_0=0 \qquad (6.5.8b)$$

Ref.[3.2] (3)

Defining an unknown X as

$$X = \frac{\omega}{\omega_h}$$

and setting the determinant of Eqs. 6.5.8 equal to zero results in a complex polynomial in X of degree four. This breaks down into the following two equations, assuming that X is always real at the flutter condition:

From the real part:

$$X^4\left(1+\frac{\rho B^4}{I}A_3^* - \frac{\rho B^2}{m}\frac{\rho B^4}{I}A_2^*H_1^* + \frac{\rho B^2}{m}\frac{\rho B^4}{I}A_1^*H_2^*\right)$$
$$+X^3\left(2\zeta_\alpha\frac{\omega_\alpha}{\omega_h}\frac{\rho B^2}{m}H_1^* + 2\zeta_h\frac{\rho B^4}{I}A_2^*\right)$$
$$+X^2\left(-\frac{\omega_\alpha^2}{\omega_h^2} - 4\zeta_h\zeta_\alpha\frac{\omega_\alpha}{\omega_h} - 1 - \frac{\rho B^4}{I}A_3^*\right)$$
$$+X\cdot 0$$
$$+\left(\frac{\omega_\alpha}{\omega_h}\right)^2 = 0 \qquad (6.5.9a)$$

Ref.[3.2] (4)

From the imaginary part:

$$X^3\left(\frac{\rho B^4}{I}A_2^*+\frac{\rho B^2}{m}H_1^*+\frac{\rho B^2}{m}\frac{\rho B^4}{I}H_1^*A_3^*-\frac{\rho B^2}{m}\frac{\rho B^4}{I}A_1^*H_3^*\right)$$
$$+X^2\left(-2\zeta_\alpha\frac{\omega_\alpha}{\omega_h}-2\zeta_h-2\zeta_h\frac{\rho B^4}{I}A_3^*\right)$$
$$+X\left(-\frac{\rho B^2}{m}H_1^*\frac{\omega_\alpha^2}{\omega_h^2}-\frac{\rho B^4}{I}A_2^*\right)$$
$$+\left(2\zeta_h\frac{\omega_\alpha^2}{\omega_h^2}+2\zeta_\alpha\frac{\omega_\alpha}{\omega_h}\right)=0 \qquad (6.5.9b)$$

These two real equations are solved successively for different assumed values of K and their roots X are plotted vs. K. At the point (X_c, K_c) where the two plots cross, the flutter condition is identified [6-66, 6-67].

The flutter problem as treated above is seen to be a semi-inverse problem since the aerodynamic coefficients are functions of the solution frequency, and a range of frequency parameters K must therefore be used to survey the solution region.

$$K_c = \frac{B\ \omega_h X_c}{U_c}$$

i.e., the critical flutter velocity is

$$U_c = \frac{B\ \omega_h X_c}{K_c} \qquad (3.26)$$

Example of Pure Torsional Flutter

Consider the case where coefficient A_2^* is the only important torsional aerodynamic coefficient, as given, for example, by Fig. 3.2, Bridge 2. Let the bridge deck mass moment of inertia per unit span be

$$I = 857{,}000 \text{ lb. sec}^2 \ (3.9 \times 10^5 \text{ kg sec}^2)$$

with $B = 100$ ft (30.5 meters). If the deck is uniform, $I_1 = C_{22} I$ and, according to eq. (3.20), the critical value of A_2^* is given by

$$(A_2^*)_{crit} = \frac{2 I \zeta_\alpha}{\rho B^4}$$

For a value of mechanical damping of $\zeta = 0.01$ and air at sea level density

$$(A_2^*)_{crit} = \frac{2,857000 \cdot (0.01)}{(0.002378) \cdot 100^4} = 0.072 \quad \text{(nondimensional)}$$

According to Fig. 3.2 this corresponds to a critical U/NB value of 7.1 which, for a natural torsional frequency of $N = 0.2$ Hz, corresponds to a flutter velocity of $U_C = 142$ ft/sec $= 96.8$ mph (= 156 km/hr).

Example of Two-Degree Flutter

The mechanical damping ζ_r and ζ_h will both be taken as 0.01; the aerodynamic coefficients will be taken from Fig. 3.3 and are as listed below (for bridge section #2). The deck width B=100 ft (30.5 m), while the span L = 4,000 ft. (1220 m). The vertical and torsional modes will be assumed to be half sine waves, therefore $C_{11} = C_{12} = C_{22} = L/2 = 2{,}000$ ft. (610 m). The constant sectional moment of inertia per unit span will be taken as I = 857,000 lb. sec^2 (3.9×10^5 kg sec^2) while the mass of the deck per unit span will be M = 711.8 lb sec^2/ft^2 (3481 kg sec^2/m^2). These lead to:

$$\left(\frac{\rho B^4}{I_1}\right)C_{11} = \left(\frac{\rho B^4}{I_1}\right) C_{12} = \left(\frac{\rho B^4}{I_1}\right)C_{22} = 0.277$$

$$\left(\frac{\rho B^2}{M_1}\right)C_{11} = \left(\frac{\rho B^2}{M_1}\right) C_{12} = \left(\frac{\rho B^2}{M_1}\right)C_{22} = 0.0334$$

It will be assumed that $\omega_\alpha = 2\,\omega_h$ and that $\omega_h = 2\pi(.1\ \text{Hz})$.

U/NB	A_1^*	A_2^*	A_3^*	H_1^*	H_2^*	H_3^*
2	0	0	0	-0.67	0	0
4	0	-0.03	0	-1.50	0	-0.05
6	0.75	-0.05	0.50	-2.05	0.7	-1.25
8	0.70	-0.10	1.00	-3.25	2.25	-3.35
10	0.68	-0.14	1.46	-4.25	4.25	-4.00
12	0.70	-0.16	1.69	-5.50	8.90	-5.00

$$b_o = 2(.01)(2)^2 + 2(.01)^2 = 0.12$$

$$b_1 = -0.1336\ H_1^* - 0.277\ A_2^*$$

$$b_2 = -0.06 - 0.00554\ A_3^*$$

$$b_3 = 0.0277\ A_2^* + 0.0334\ H_1^* + .00925\ [H_1^* A_3^* - A_1^* H_3^*]$$

<u>U/NB = 12</u> (K = .523)

$$b_o = .12$$

$$b_1 = (-.1336)(-5.50) - 0.277(-.16) = .7791$$

$$b_2 = -.06 - 0.00554(1.69) = -.06936$$

$$b_3 = (.277)(-.16) + [.0334)(-.5.5)$$

$$+ .00925[(-5.5)(1.69) + 5(.70)]$$

$$= -.2816$$

$$(-.2816)X^3 - (.06936)X^2 + (.7791)X + .12 = 0$$

$$X^3 + (.2462)X^2 - 2.7667X - .4261 = 0$$

$$X = 1.62$$

(Negative roots neglected)

$\underline{U/NB = 10}$ ($\dot{K}$ = .628)

$$b_o = .12$$

$$b_1 = (-.1336)(-4.25) - (.277)(-.14) = 0.6066$$

$$b_2 = -.06 - .00554(1.46) = -.06809$$

$$b_3 = (.277)(-.14) + .0334\ (-4.25) + .00925\ [(-4.25)(1.46) + 4(.68)] = -.2130$$

$$(-.2130)X^3 - (.06809)X^2 + (.6066)X + .12 = 0$$

$$X^3 + .3197\ X^2 - 2.848\ X - .5634 = 0$$

$$X = 1.63$$

$\underline{U/NB = 8}$ $(K = .785)$

$$b_0 = .12$$

$$b_1 = (-.1336)(-3.25) - .277(-.10) = 0.4619$$

$$b_2 = -.06 - (.00554)(1.00) = -.06554$$

$$b_3 = (.277)(-.10) + (.0334)(-3.25)$$

$$+ .00925\,[(-3.25) + 3.35(.70)]$$

$$= -.1446$$

$$(-.1446)X^3 - (.06554)X^2 + .4619\,X + .12 = 0$$

$$X^3 + (.4533)X^2 - (3.194)X - .8299 = 0$$

$$X = 1.71$$

$\underline{U/NB = 6}$ (K = 1.05)

$$b_0 = .12$$

$$b_1 = (-.1336)(-2.05) - (.277)(-.05) = .2877$$

$$b_2 = -.06 - (.00554)(0.50) = -.06277$$

$$b_3 = (.277)(-.05) + (.0334)(-2.05)$$

$$+ .00925 [(-2.05)(.50) - (.75)(-1.25)]$$

$$= -0.0831$$

$$(-.0831)X^3 - (.06277)X^2 + (.2877)X + .12 = 0$$

$$X^3 + (.7554)X^2 - 3.462 X - 1.444 = 0$$

$$X = 1.73$$

$\underline{U/NB = 4}$ $(K = 1.57)$

$$b_0 = .12$$

$$b_1 = (-.1336)(-1.50) - .277(-.03) = .2087$$

$$b_2 = -.06 - .0054(0) = -.06$$

$$b_3 = -.00831 + .0334(-1.5) = -.0584$$

$$(-.0584)X^3 - (.06)X^2 + (.2087)X + .12 = 0$$

$$X^3 + (1.027)X^2 - (3.574)X - 2.055 = 0$$

$$X = 1.73$$

$\underline{U/NB = 2}$ $(K = 3.14)$

$$b_0 = .12$$

$$b_1 = (-.1336)(-0.67) - (.277)(0) = .08951$$

$$b_2 = -.06$$

$$b_3 = (.0334)(-.67) = -.02238$$

$$(-.02238)X^3 - (.06)X^2 + (.08951)X + .12 = 0$$

$$X^3 + 2.681X^2 - 4.00X - 5.362 = 0$$

$$X = 1.63$$

Quartic Equation

$$a_0 = (2)^2 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008 - .277\ A_3^*$$

$$a_3 = .00134\ H_1^* + .00554\ A_2^*$$

$$a_4 = 1 + .277\ A_3^* + .00953\ [A_1^*\ H_2^* - A_2^*\ H_1^*]$$

<u>U/NB = 12</u> (K = .523)

$$a_0 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008 - (2.77)(1.69) = -5.469$$

$$a_3 = (.00134)(-5.50) + (.00554)(-.16) = -.00826 \text{ (neglig.)}$$

$$a_4 = 1 + (.277)(1.69) + .00953\ [(.70)(8.9) - 5.50(.16)]$$

$$= 1.519$$

$$(1.519)X^4 - 5.469X^2 + 4 = 0$$

$$X^2 = \frac{5.469 \pm \sqrt{(5.469)^2 - 4(4)(1.519)}}{2(1.519)}$$

$$X^2 = 2.579 \;, \quad 1.021$$

$$X = 1.61 \;, \quad 1.01$$

U/NB = 10 (K = .628)

$$a_0 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008 - (.277)(1.46) = -5.405$$

$$a_3 = \text{negligible}$$

$$a_4 \quad 1 + (.277)(1.46) + .00953\,[(.68)(4.25) - (4.25)(.14)]$$

$$= 1.426$$

$$1.426\,X^4 - 5.405\,X^2 + 4 = 0$$

$$X^2 = \frac{5.405 \pm\sqrt{(5.405)^2 - 4(4)(1.426)}}{2(1.426)}$$

$$X^2 = 2.782 \;,\; 1.008$$

$$X = 1.67 \;,\; 1.00$$

$\underline{U/NB = 8}$ $(K = .785)$

$$a_0 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008 - (.277)(1.00) = -5.278$$

$$a_3 = \text{negligible}$$

$$a_4 = 1 + (.277)(1.00) + .00953\,[(.70)(2.25) - (.10)(3.25)]$$

$$= 1.289$$

$$1.289\,X^4 - 5.278\,X^2 + 4 = 0$$

$$X^2 = \frac{5.278 \pm \sqrt{(5.278)^2 - 4(4)(1.289)}}{2(1.289)}$$

$$X^2 = 3.091 \quad , \quad 1.004$$

$$X = 1.76 \quad , \quad 1.00$$

$\underline{U/NB \quad = \quad 6}$ $\quad (K = 1.05)$

$$a_0 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008 - .277(.50) = -5.139$$

$$a_3 = \text{negligible}$$

$$a_4 = 1.+ (.277)(.50) + .00953 [(.75)(.70) - 2.05 (.05)]$$

$$= 1.143$$

$$(1.143)\, X^4 - 5.139\, X^2 + 4 = 0$$

$$X^2 = \frac{5.139 \pm \sqrt{(5.139)^2 - 4(4)(1.43)}}{2(1.143)}$$

$$X^2 = 3.495 \quad , \quad 1.001$$

$$X = 1.87 \quad , \quad 1.00$$

U/NB = 4 (K = 1.57)

$$a_0 = 4$$

$$a_1 = 0$$

$$a_2 = -5.0008$$

$$a_3 = \text{negligible}$$

$$a_4 = 1.000$$

$$X^2 = \frac{5.0008 \pm \sqrt{(5.0008)^2 - 16}}{2}$$

$$X^2 = 4.001 \ , \ .99973$$

$$X = 2.000 \ , \ 1.000$$

$\underline{U/NB = 2.0}$ $(K = 3.14)$

$$a_0 = 4$$

$$a_1 = 0$$

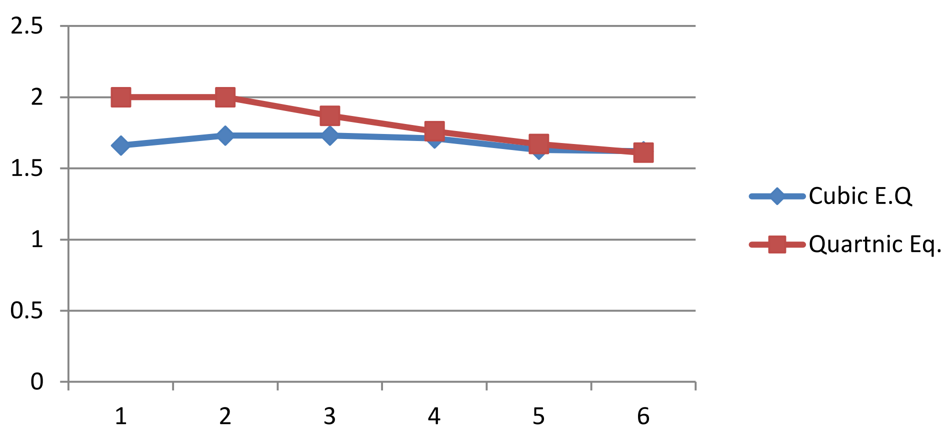
$$a_2 = -5.0008$$

$$a_4 = 1.00$$

$$X = 2.000 \;,\; 1.000$$

VALUES OF $X = \frac{\omega}{\omega h}$

U/NB	K	Cubic Eq.	Quartic Eq.	
2	3.14	1.66	1.00	2.00
4	1.57	1.73	1.00	2.00
6	1.05	1.73	1.00	1.87
8	.785	1.71	1.00	1.76
10	.628	1.63	1.00	1.67
12	.523	1.62	1.01	1.61



Cubic equation intersects quartic equation at

$$X = 1.62 \qquad K = .53 \qquad (U/NB = 11.7)$$

Therefore,

$$U_c = \frac{B\omega_h X_c}{K_c}$$

$$U_c = \frac{(100\text{ ft})\,[2\pi\,(.1)]\,1.62}{0.53}$$

$$U_c = 190.3\text{ ft/sec} = 129.7\text{ mph}$$

(58.0 m/sec)

Conclusions

- Scanlan's method is easy and accurate.
- Flutter derivatives are main roles for flutter analysis.
- Therefore to obtain flutter derivatives by wind tunnel tests are very important.
- Bridge designers should practice flutter analysis by hands at least one time.

References

[3.1] Scanlan, R.H.: "Recent Methods in the Application of Test Results to the Wind Design of Long, Suspended-Span Bridges," Report No. FHWA-RD-75-115, Federal Highway Administration Office of Research and Development, Washington, DC, 1975.

[3.2] Simiu, E. and Scanlan, R.H.: Wind Effects on Structures, Wiley, New York, 1978.

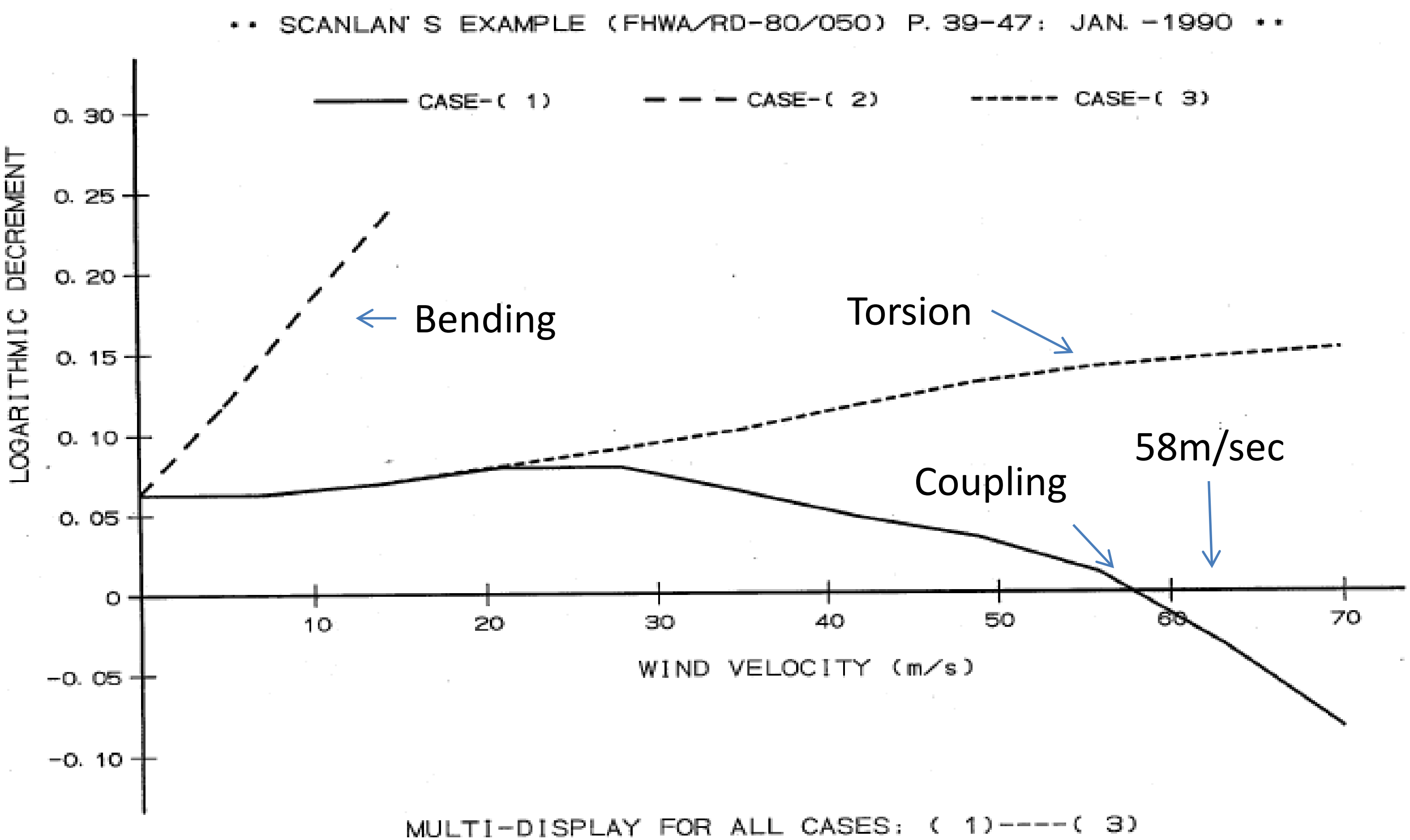
[3.3] Scanlan, R.H. and Tomko, J.J.: "Airfoil and Bridge Deck Flutter Derivatives," Jnl. Eng. Mech. Div. ASCE, Vol. 97, No. EM6, Dec. 1971, pp. 1717-1737.

PART－2

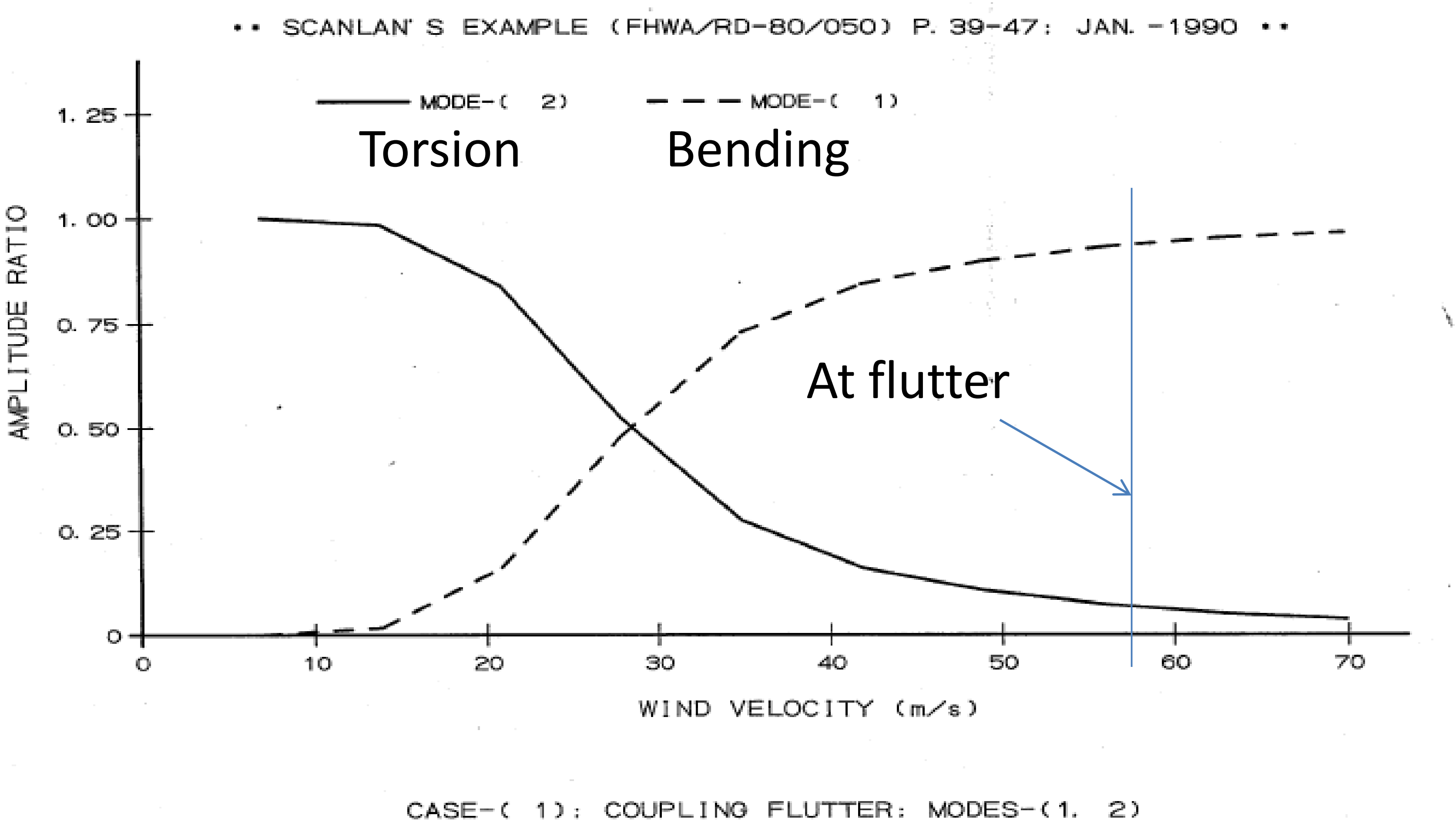
- Flutter analysis method will be applied to same problem.
- ・ The method was developed by Hiroshi TANAKA at Hitachi Zosen in 1990.
- ・ He received 'Tanaka Prize' by the following paper.

Paper
Multi-Mode Flutter Analysis and
Two & Three Dimensional Model Tests on
Bridges with Non-analogous Modal Shapes
JSCE J.Struct.Mech.Earthquake Eng. July 1993

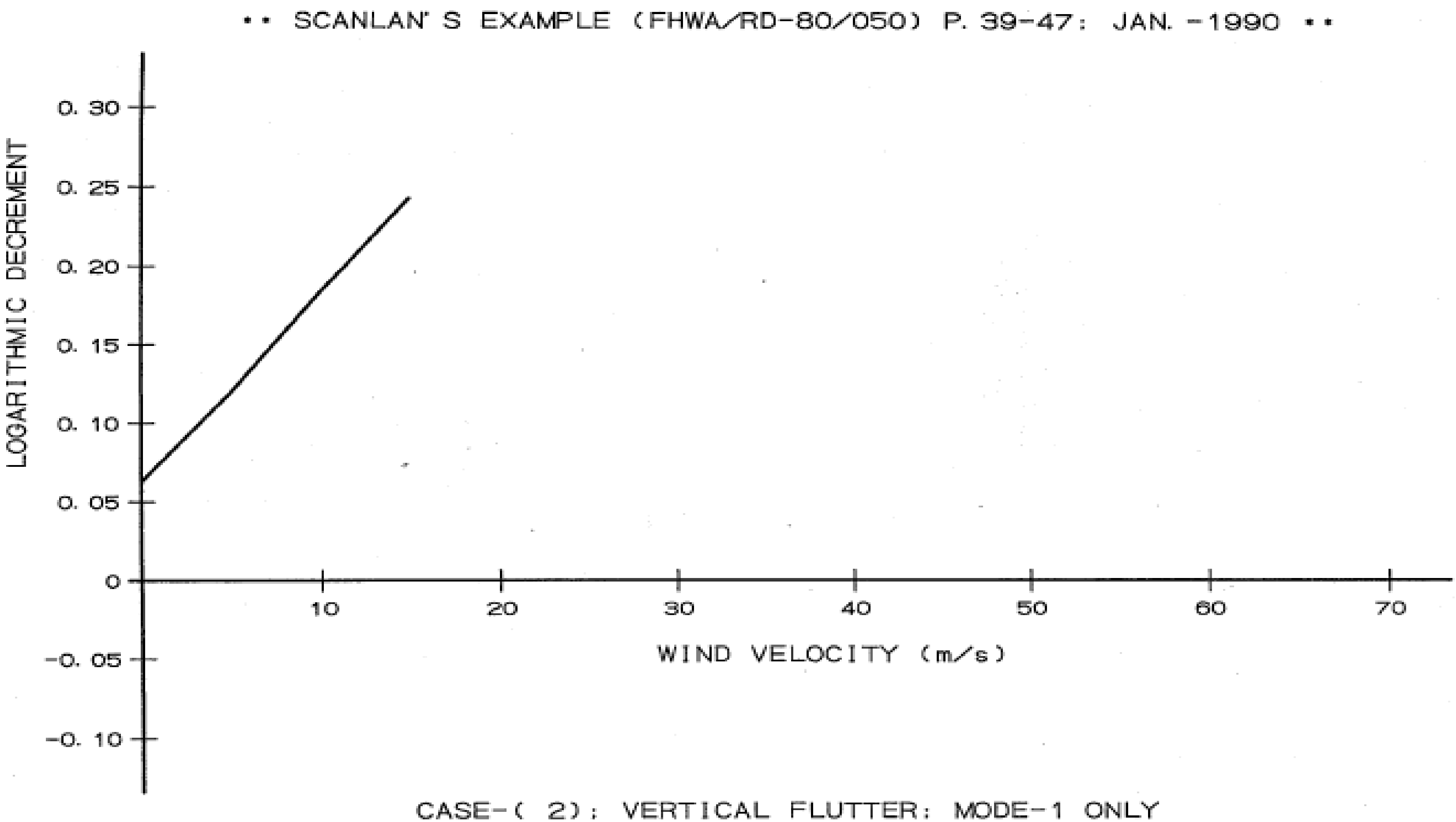
Results of Dr. Tanaka



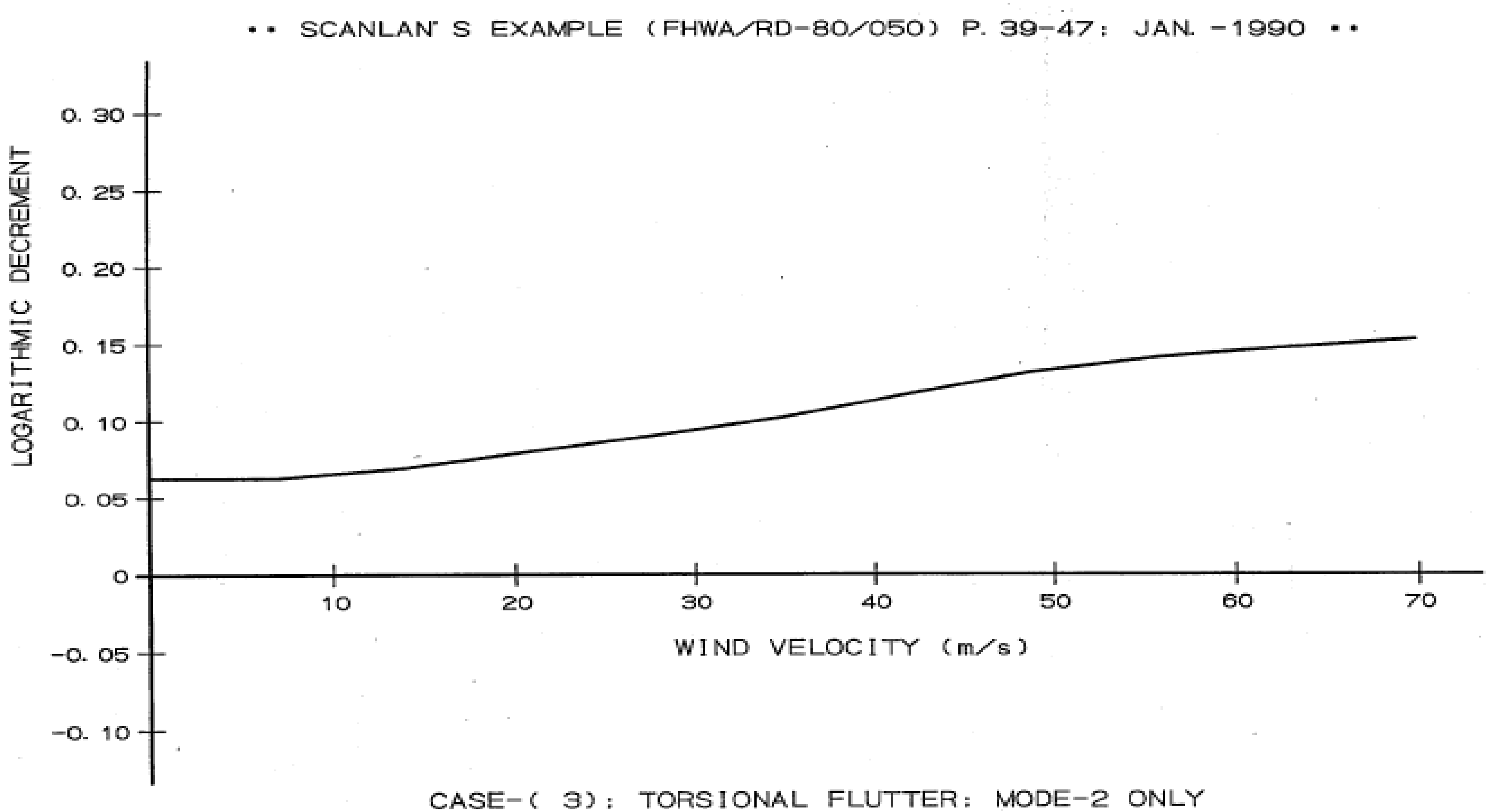
Amplitude Ratio of mode-1 and 2



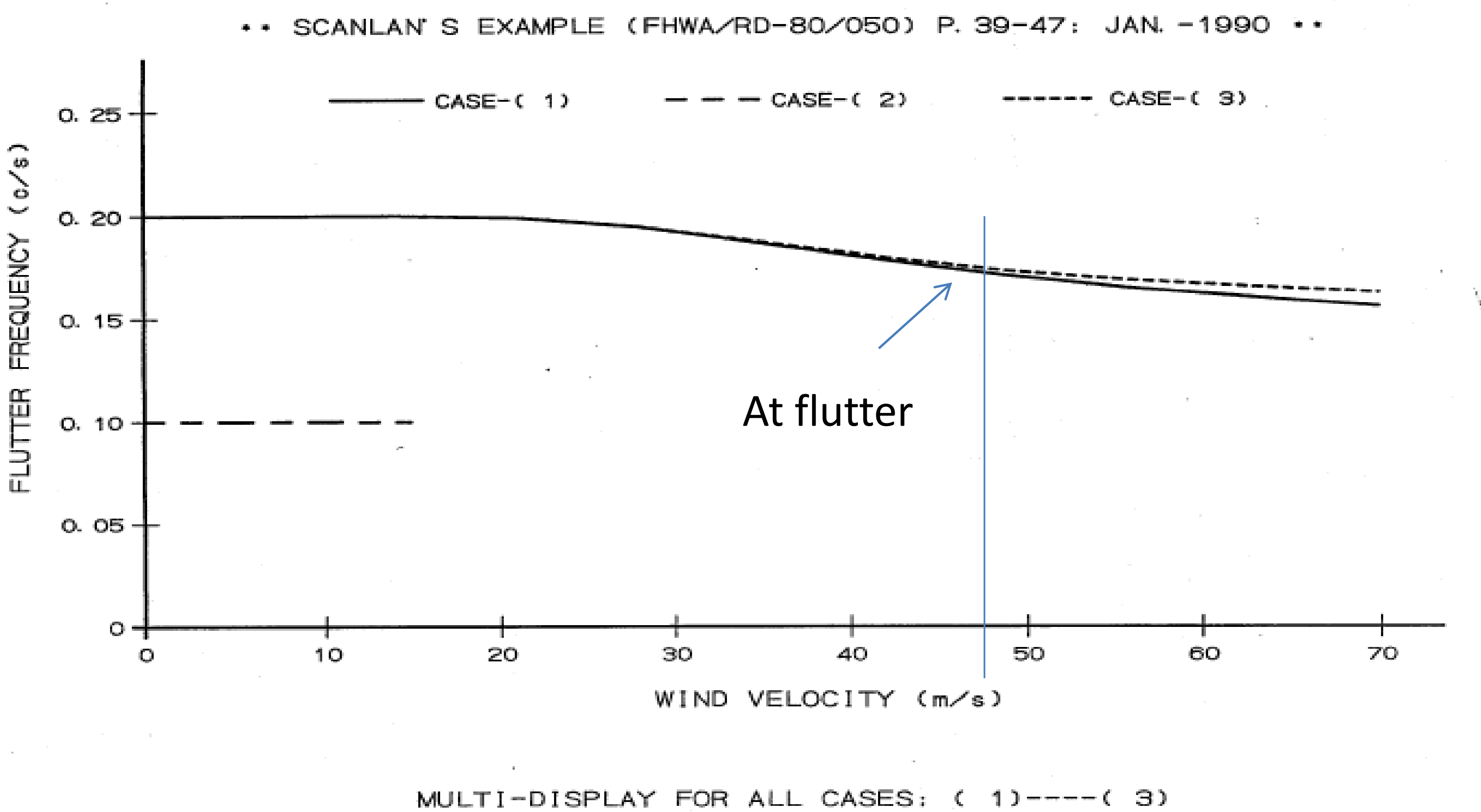
Change of Damping (Bending)



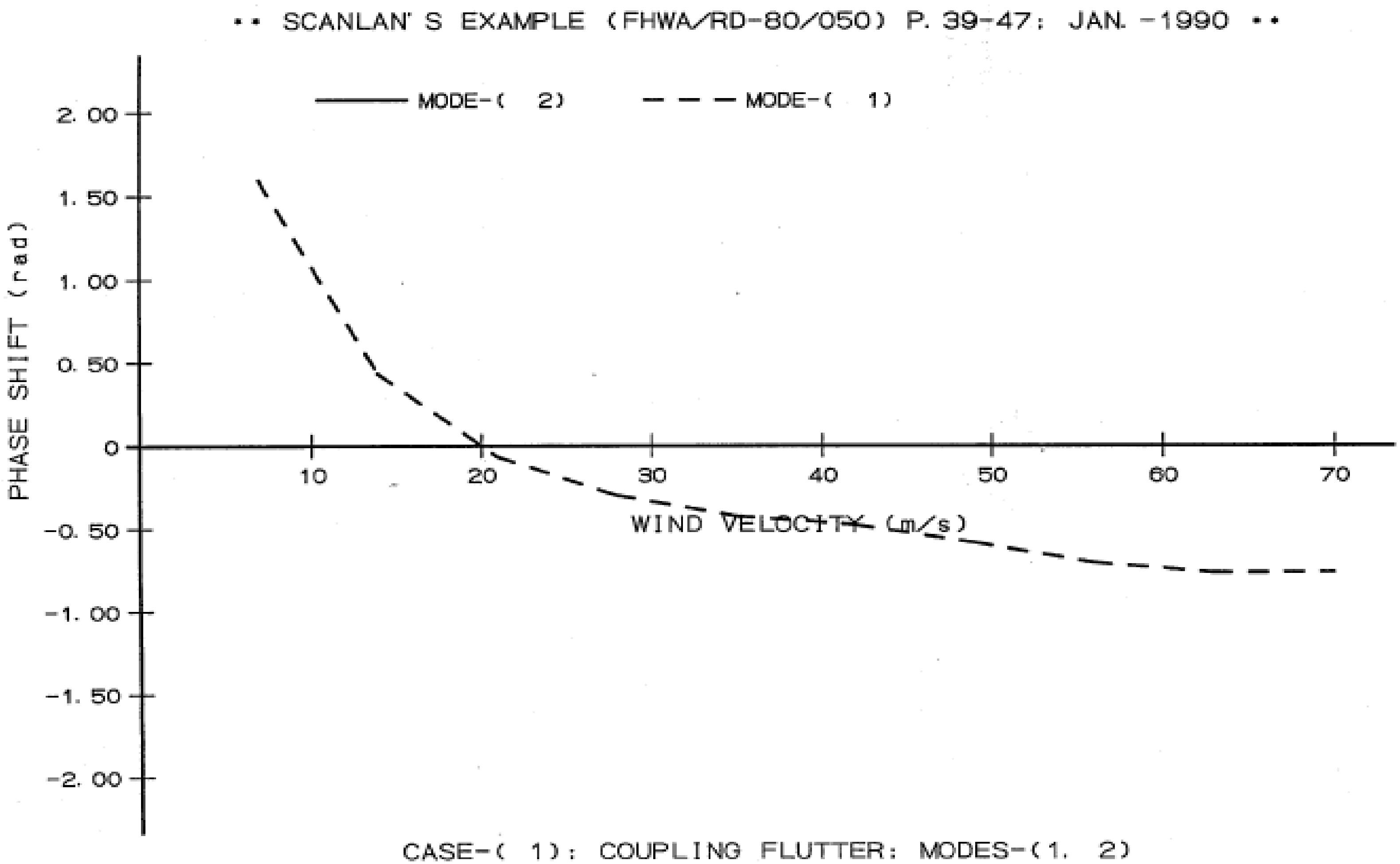
Change of Damping (Torsion)



Flutter Frequency



Phase Shift (rad)



Input Form

PAGE = 1

```
          1         2         3         4         5         6         7         8
....+....0....+....0....+....0....+....0....+....0....+....0....+....0....+....0
TITL ** SCANLAN'S EXAMPLE (FHWA/RD-80/050) P.39-47; JAN.-1990 **
CNTL    2    98        1    1    1
UNIT    1
MATL    1   20.5800   79.3800
SECT    1  334.2780    1        1488          1         115609   37455.60
NODE    1              0                              1 1 1 1
NODE    2            122
NODE    3            244
NODE    4            366
NODE    5            488
NODE    6            610
NODE    7            732
NODE    8            854
NODE    9            976
NODE   10           1098
NODE   11           1220                              1 1 1 1
MEMB    1    1    2    1    1                           0.0
MEMB    2    2    3    1    1                           0.0
MEMB    3    3    4    1    1                           0.0
MEMB    4    4    5    1    1                           0.0
MEMB    5    5    6    1    1                           0.0
MEMB    6    6    7    1    1                           0.0
MEMB    7    7    8    1    1                           0.0
MEMB    8    8    9    1    1                           0.0
MEMB    9    9   10    1    1                           0.0
MEMB   10   10   11    1    1                           0.0
CLAS .0628    1   11
SELF    1   10    1
HST1  2.0   -1.34  4.0   -3.00  6.0   -4.10  8.0   -6.50  10.0   -8.50
HST1 12.0  -11.00
HST2  2.0    0.00  4.0    0.00  6.0   -1.40  8.0   -4.50  10.0   -8.50
HST2 12.0  -17.80
HST3  2.0    0.00  4.0    0.10  6.0    2.50  8.0    6.70  10.0    8.00
HST3 12.0   10.00
AST1  2.0    0.00  4.0    0.00  6.00  -1.50  8.0   -1.40  10.0   -1.36
AST1 12.0   -1.40
AST2  2.0    0.00  4.0   -0.06  6.00  -0.10  8.0   -0.20  10.0   -0.28
AST2 12.0   -0.32
AST3  2.0    0.0   4.0    0.0   6.0    1.00  8.0    2.00  10.0    2.92
AST3 12.0    3.38
FLUT     1COUPLING FLUTTER; MODES-(1, 2)
FACT   1.22794  50        9999.
COPL    5  70.0       10         2    1                    1
LOD2    1   10    1    1                      0.00001   30.5
FLUT     2VERTICAL FLUTTER; MODE-1 ONLY
FACT   1.22794  50        9999.
COPL    5  15.0        3         1                         1
LOD2    1   10    1    1                      0.00001   30.5
FLUT     3TORSIONAL FLUTTER; MODE-2 ONLY
FACT   1.22794  50        9999.
          1         2         3         4         5         6         7         8
....+....0....+....0....+....0....+....0....+....0....+....0....+....0....+....0
```

PAGE = 2

```
          1         2         3         4         5         6         7         8
....+....0....+....0....+....0....+....0....+....0....+....0....+....0....+....0
COPL    5  70.0       10         2                         1
LOD2    1   10    1    1                      0.00001   30.5
END
          1         2         3         4         5         6         7         8
....+....0....+....0....+....0....+....0....+....0....+....0....+....0....+....0
```

Conclusions

- Tanaka Method will give you many information on flutter.
- Input is small.
- Calculation time is less than 10 sec.
- You can understand flutter phenomenon.

THANK YOU !!